Coordinating Analyses of Tunings with Analyses of Pieces

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ANALYSES of tunings have often been carried out independently of pieces in which they are actually realized. Whereas tunings are prima facie relevant to pieces in which they occur, to what extent is this so? And does such a relationship hold in both directions? That is, are analyses of pieces relevant to analyses of their tunings?

Both sorts of analysis involve methodological problems and, at least in principle, both sorts of analysis should mesh. Germane to the present discussion are instances of such analytical problems that arise in Central Javanese *pélog* tunings and in "skeletal melodies" (*balungans*) of multi-section pieces (*gendhings*) that employ these tunings.

The present account identifies such problems and proposes solutions that attempt to coordinate both sorts of analysis. With regard to tuning per se, relationships among acoustical spectra, pitch determinacy, and interval categorization are considered. Concerning individual pieces, both jointly and severally, longstanding notions about "exchange," "shifting," "alternate," or "substitute" tones (*sorogan*), modal identity (*pathet*), and gong tones are addressed. Linking both kinds of analysis—and shared by both—is an expanded formulation of Wertheimer's Gestalt Grouping Principle of Similarity. Introduced from post-tonal analysis of European-derived music are concepts of common tones, "well-formed" (WF) scales, and interval vectors.

In what follows, I focus on three topics:

- Tones produced by two kinds of instruments, called *sarons*, that are employed in traditional music of Central Java: the names for these kinds of instruments are *saron demung* and *saron barung*.
- The scale or scales that result from these tones: in particular, scales that result from tones produced by sarons that are in so-called pélog tuning.
- An important part of many pieces in pélog tuning that is played by the sarons, namely, the balungan of a piece: English-language translations of "balungan" have been "skeletal melody," "core melody," and "nuclear theme."

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Saron Tones

Below is a photograph of a saron in the gamelan Kyai Parijata (discussed below). Each saron has seven metal keys or bars. When struck with a wooden hammer or mallet, the seven metal keys of a saron produce seven tones. Each of the seven keys has a traditional Central Javanese name. As well, each of the seven keys has been identified with a number: from left to right in the photograph, these numbers are ordered from 1 to 7.

Source: https://web.archive.org/web/20150823035024/http://www.marsudiraras.org/gamelan/sarondemung_photo_sound.html

In a traditional Central Javanese ensemble, there are generally two saron parts. Each of these two parts might be played by a single saron or by more than one saron. If more than one saron plays a particular part, all the instruments playing that part strike the same numbered key at the same time. In general, for both saron parts, players strike the same numbered key at the same time: For example, key 1 is struck at the same time by the players of both the saron parts; similarly for the keys numbered 2 to 7.

The sarons on which I focus are part of an ensemble whose name is Kyai Parijata. Kyai Parijata is a set of instruments that are approximately 200 years old (Heins 1968–69). This set of instruments, or *gamelan*, has been housed at the Museum of Ethnography Nasuntara in Delft, the Netherlands. Until three years ago, members of the gamelan club Marsudi Raras in Delft played on the museum's instruments, and the Marsudi Raras club maintained its website until recently (Timer4web 2016).

Marsudi Raras's website has been especially valuable to researchers. In addition to information about the club's activities and recordings of some of its concerts and rehearsals, the website has provided very carefully recorded .wav files of individual tones produced by Kyai Parijata's instruments (Oldenborgh 2002). These .wav files could be readily downloaded and might, as the club's website said, be redistributed freely, according to a Creative Commons license. Because the Marsudi Raras club has recorded individual .wav files of Kyai Parijata's tones one can hear what the tones produced by the ensemble's two sarons sound like. From left to right, the 14 tones sound like this.

In the present report, I distinguish between acoustical properties of individual tones produced by sarons, which I regard as components of the instruments' tuning, and relationships one can, in principle, hear among such tones in actual pieces, which I consider aspects of a scale or scales. How, then, can one analyze acoustical features of sarons, that is, their tuning, with a view to understanding perceptually the tones they produce as a scale or scales?

Acoustically, each tone produced by sarons comprises one or more partials, and if there is more than one partial, these partials are inharmonic. That is, unlike harmonic partials of tones produced by the human voice or by violins, woodwinds, or brass instruments, the inharmonic partials produced by sarons do not correspond to the overtone series. The pitch of a tone that has harmonic partials corresponds to the frequency of the lowest tone in its overtone series. For saron tones whose spectra are inharmonic, researchers have generally identified the tone's perceived pitch with the frequency of the partial whose amplitude is greatest. However, systematic musicologist Albrecht Schneider (1991, 2001) has called into question this approach. According to Schneider, the partials of saron tones have very complex, unstable vibration patterns, and no stationary or quasi-stationary portion, which he regards as a prerequisite for reliable "pitch" judgments.

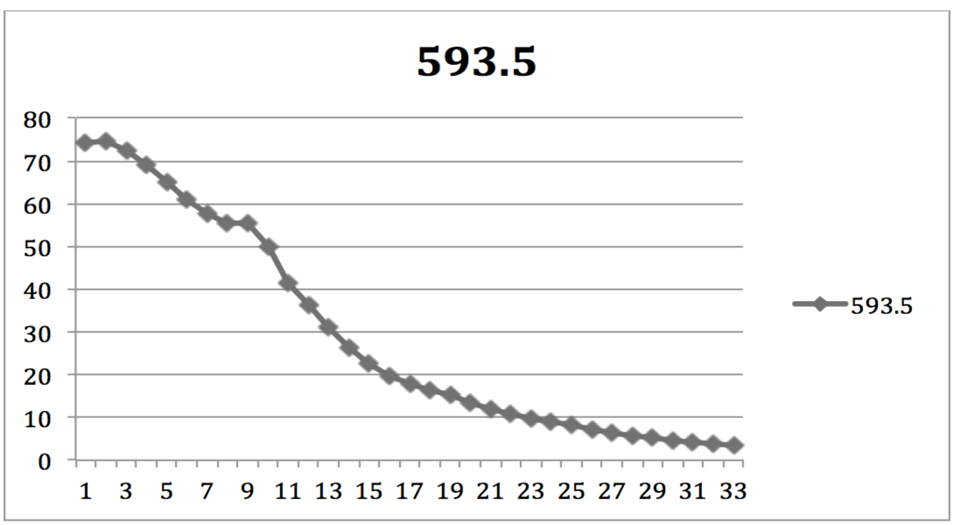
For Schneider, the spectral content of a saron tone shifts as a function of time. Accordingly, he has claimed that there are marked shifts in "pitch" in such a tone, so that the auditory impression is that of fluctuation and "uncertainty." Indeed, Schneider has said that the pitches of saron tones are "often" uncertain. Other terms he has used are vague and ambiguous.

On one hand, publications where Schneider has made such claims have been widely treated as authoritative: e.g., Google Scholar currently lists more than 50 publications that have cited this research. On the other hand, Schneider's (1991, 308) only published evidence for pitch shifts produced by sarons is a spectral analysis of a single saron tone in waterfall format. In other words, Schneider considers the pitch perception of a tone to be a direct counterpart to its acoustical character.

Taking to heart Schneider's discussion, I analyzed the spectra of Kyai Parijata's saron tones. As a first step in identifying the pitches produced by the saron tones of Kyai Parijata, I undertook spectral analyses. Spectral analyses of the individual tones were greatly facilitated by Wavanal software that Bill Hibbert (2011) developed a few years ago. Like the Marsudi Raras website, Wavanal software is of considerable value to researchers, especially as it is both easy to run and readily downloadable online, at no cost.

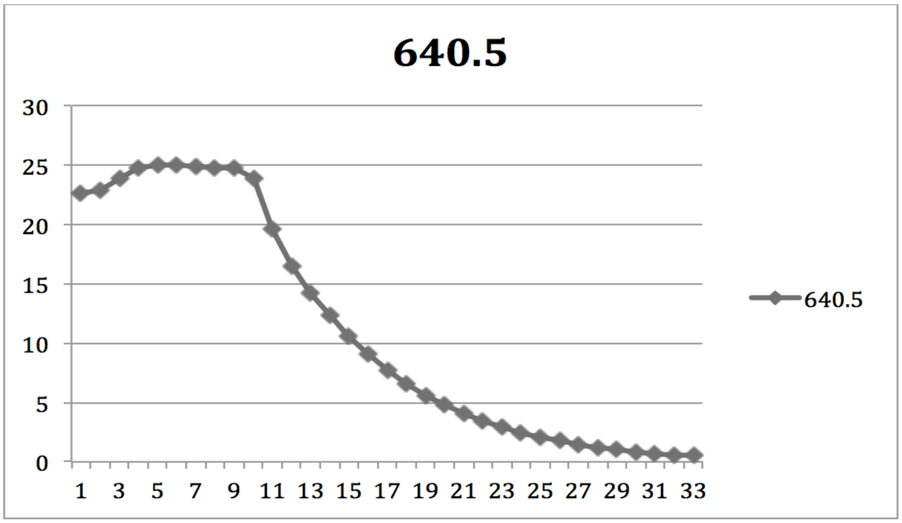
The spectral analyses of Kyai Parijata's saron barung tones were remarkably simple and uniform. For each saron barung tone, Wavanal identified the frequency of only one partial. Moreover, the loudness of the single frequency, which Wavanal conveys in perceptually relevant phons rather than in decibels, declined soon after the onset, and continually to the end of the tone.

In the following seven graphs, time is indicated from left to right: each of the first eight increments spans 12.5 ms (i.e., milliseconds), for 100 ms (one tenth of a second) in total; the remaining increments correspond to 200 ms each, for a total duration of as much as five seconds. Loudness is indicated from low to high: e.g., the first tone begins at more than 70 phons and concludes close to zero phons. The frequency of the only partial Wavanal identified for each tone appears at the top of the graph. For example, for the tone analyzed in the first image (key 1), Wavanal identified the frequency as 593.5 Hz, i.e., 593.5 cycles per second.[1]

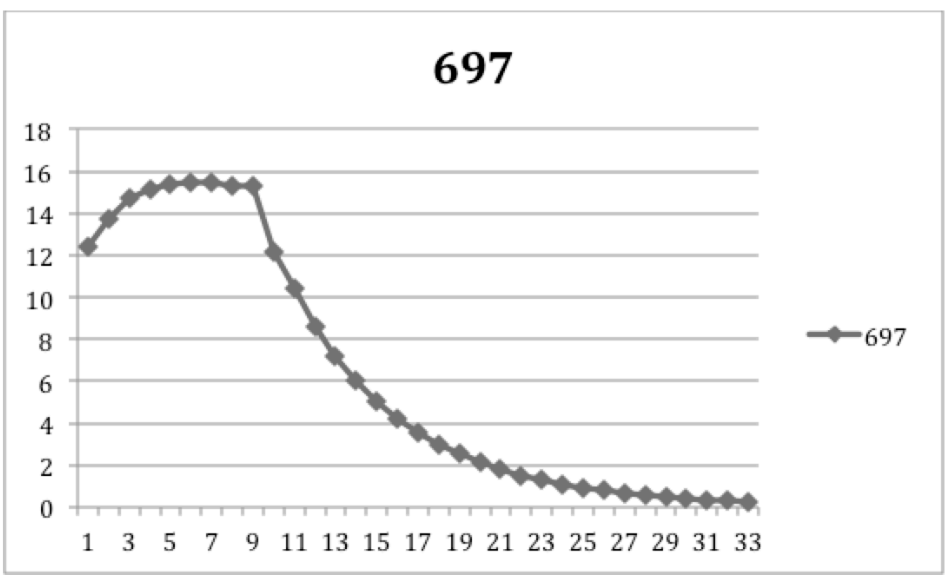


Saron barung, key 1.

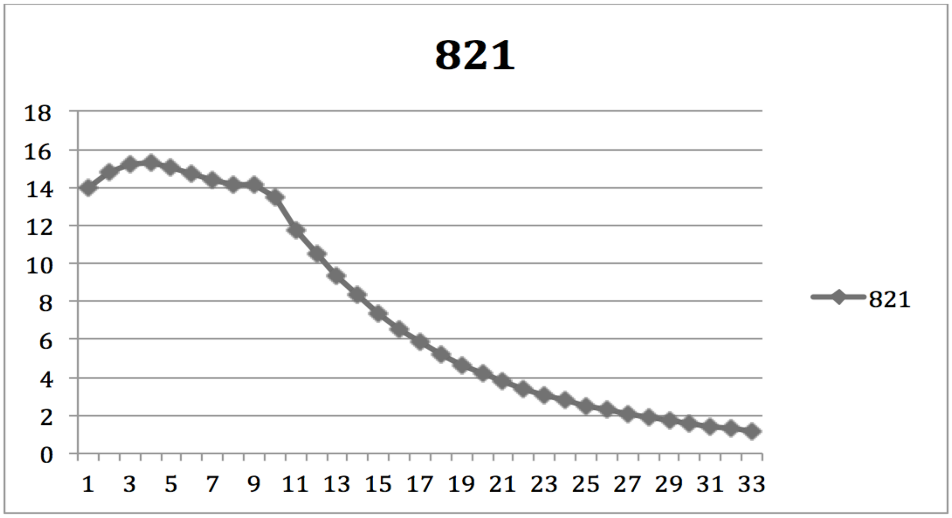
[1] Note that Wavanal measures frequencies to the nearest half-cycle per second.



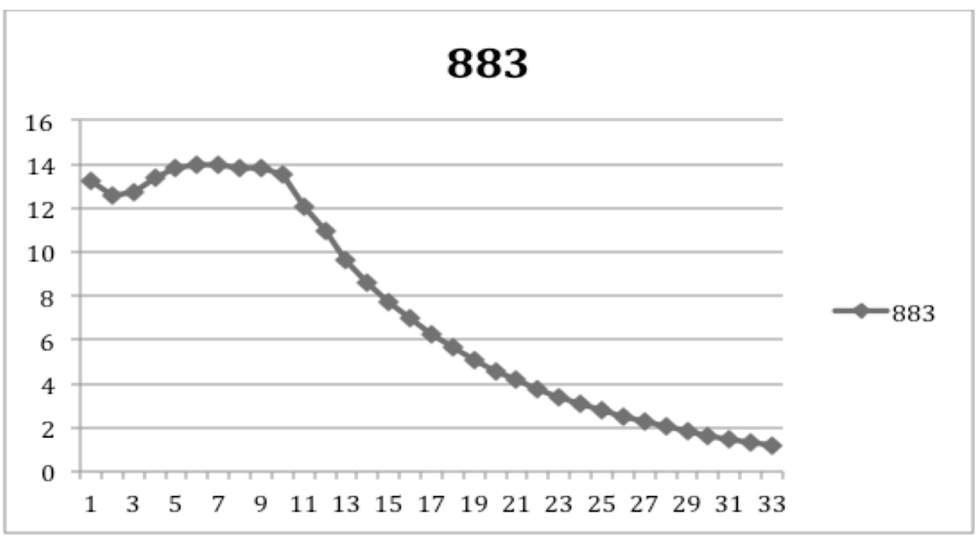
Saron barung, key 2.



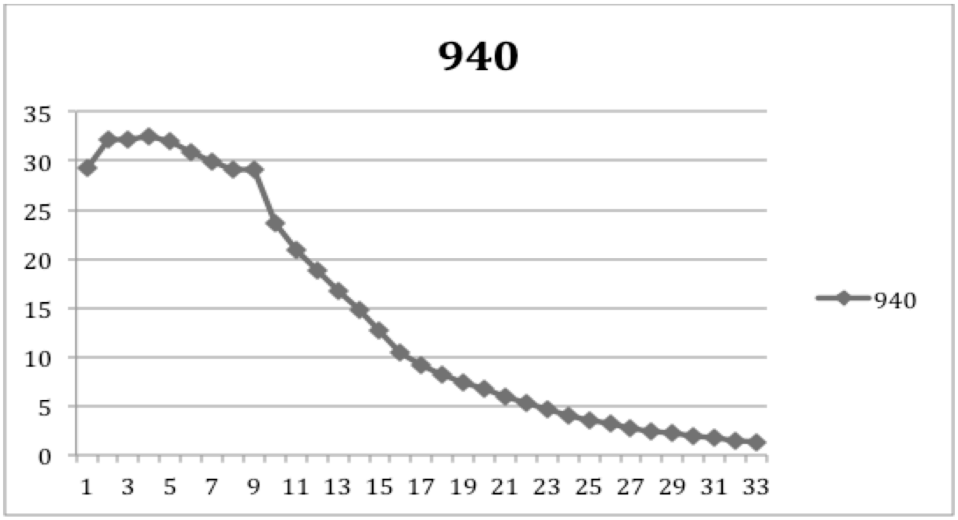
Saron barung, key 3.



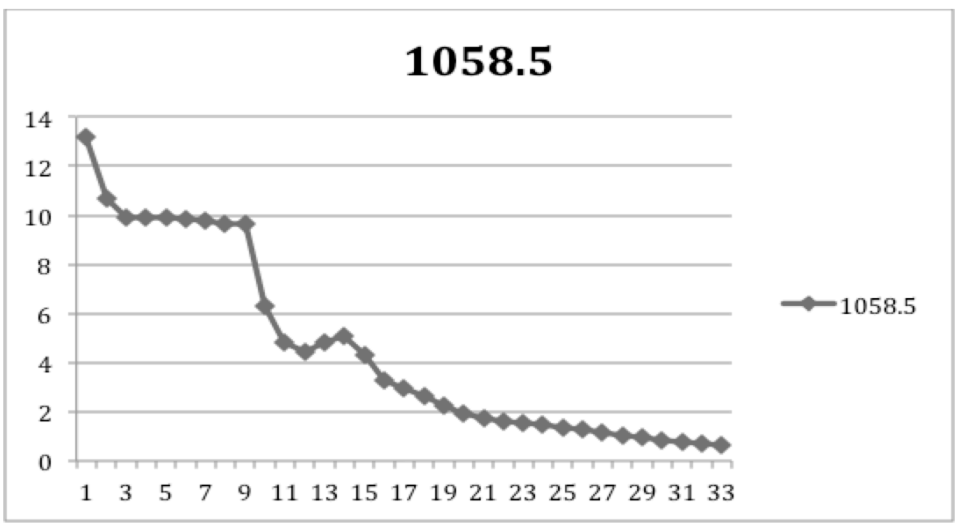
Saron barung, key 4.



Saron barung, key 5.



Saron barung, key 6.

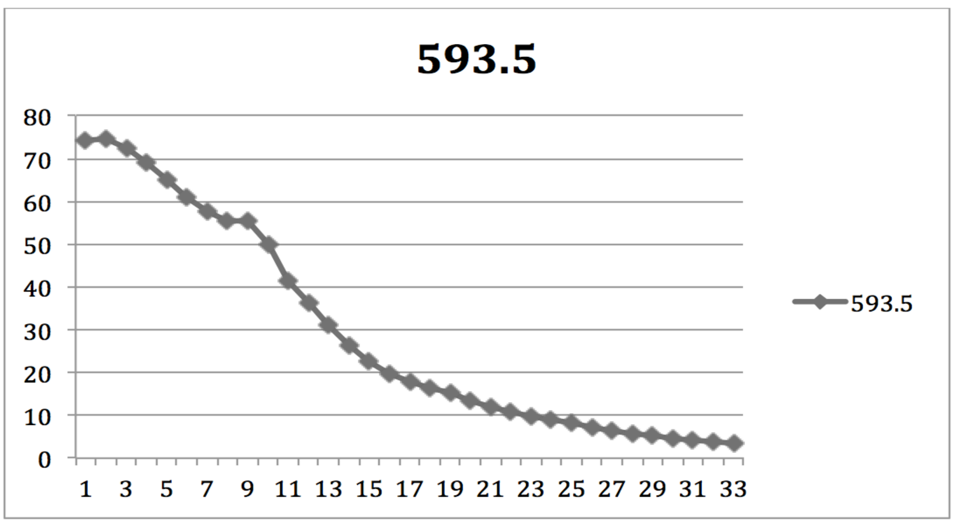


Saron barung, key 7.

A standard way of identifying "the pitch" of a tone is to generate a sine tone that is heard as matching, in pitch, the tone in question (Hartmann 1997, 283–84). As a preliminary step in identifying their pitches, I generated sine tones that had the same frequency as the loudest frequency in each of the tones' respective spectra. Generating these sine tones was greatly facilitated by Audacity software (2008–16). Like Wavanal, Audacity is both easy to run and readily downloadable online at no cost.

A perceptual phenomenon that recurred within the saron barung tones was a rise in pitch. One might be tempted to attribute this rise in perceived pitch to a corresponding change in acoustically measured frequency. However, as acoustician, William M. Hartmann (1978) reported almost 40 years ago, a sine tone that declines in amplitude is heard as rising in pitch even though its physical frequency does not change (see also Savage et al. 1977).

Whereas the sine tones initially generated to determine the tones' pitches remained uniform in amplitude throughout, the saron barung tones declined in amplitude. One can compare the two kinds of tones in this audio example: first a sine tone of 593.5 Hz, then the saron tone in the graph below, whose sole frequency is 593.5 Hz, then the 593.5-Hz sine tone again.[2] If your hearing is like the hearing of others, the beginning of the second tone, produced by the saron barung, will be heard as the same in pitch as the first tone, a sine tone, but the end of the second, barung tone will sound higher than the third tone, which is the sine tone again, because of the saron tone's rise in perceived pitch.



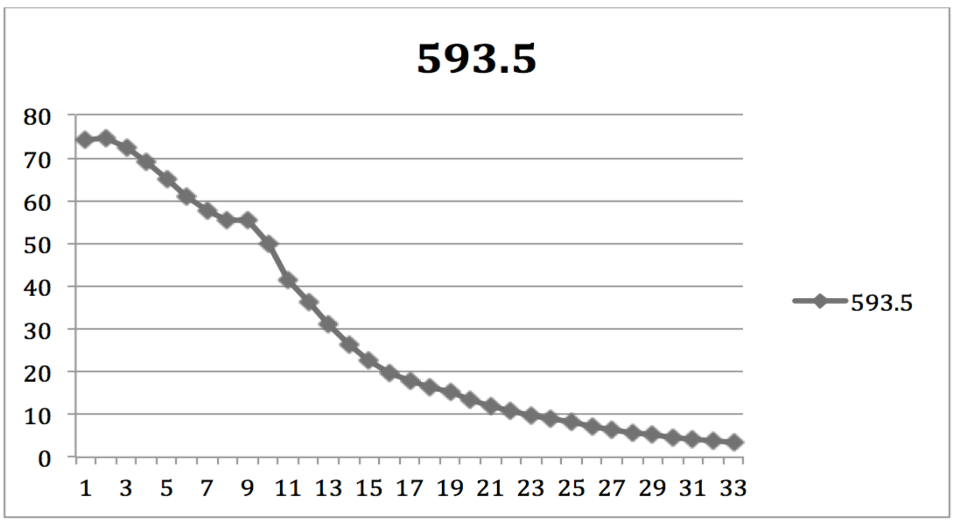
Saron barung, key 1.

As a consequence of both this phenomenon and Wavanal's spectral analysis, one can conclude with confidence that, after their often noisy onsets (which Hibbert [2011] terms their "splashes"), the saron barung tones are effectively sine tones that acoustically decline in amplitude and perceptually decline in loudness while

[2] To optimize direct comparisons of the sounds they produce, the audio files should be played with suitable amplification and an adequate speaker or headphones, rather than merely, e.g., through the internal speaker of a laptop.

perceptually ascending in pitch even though they do not change acoustically in frequency. Effectively, they sound like very large tuning forks.

A second consequence was that I decided to determine the pitches of such saron tones by comparing sine tones with only an initial portion of the saron tones' durations. As the aim of the study was to relate perception of individual tones ultimately to audible aspects of entire pieces, focusing on the initial portion of tones was further justified. This is because the beginning of a saron tone, i.e., immediately after its initial, noisy splash, is its most salient part within the context of a much larger passage or the entire piece in which it is heard. Accordingly, I compared, in immediate succession, the first 500 ms of each saron barung tone in alternation with a 500-ms sine tone having the same frequency, as this audio example illustrates, again for the sine and saron tones of 593.5 Hz.



Saron barung, key 1.

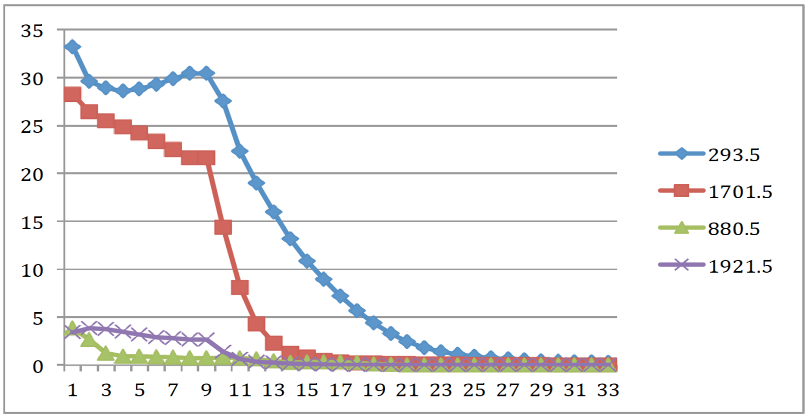
As I perceived no difference in pitch between the two kinds of tones, I entered into an Excel spreadsheet each sine tone's acoustically determined frequency as a surrogate value for the perceived "pitch" of the corresponding saron barung tone. These surrogate values were a basis for concluding that the pitches of tones 1 to 7—as I had been rather certain of beforehand—were ordered, respectively, from lowest to highest. These surrogate values were also a basis for subsequently considering pairs of tones to be heard as larger or smaller pitch-intervallically, i.e., with regard to their pitch-intervallic magnitudes.

At this point, I shall introduce a set of notions that have guided each stage of the present research. To illustrate, one can consider the following visual–spatial display to be analogous to the successions of four tones you just heard.

From left to right, the first and third, unfilled circles correspond to the sine tones, and the second and fourth, filled circles correspond to the saron barung tones. Two of the circles differ from the other two in filledness; two of the tones differ from the other two in timbre or tone quality. However, all four circles are seen as the same in shape, size, and horizontal orientation, and all four tones are heard as the same in pitch. According to Max Wertheimer's (1923) Gestalt Grouping Principle of Similarity, all four *circles* would be seen as a group with regard to shape, size, and location on a y-axis, and all four tones would be heard as a group with regard to pitch.

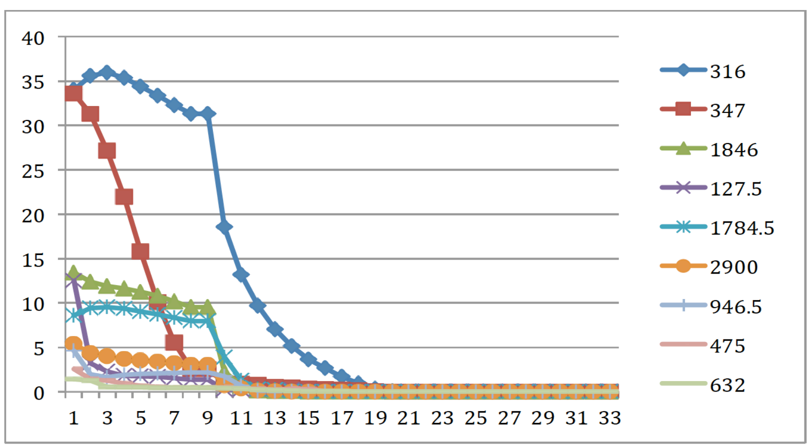
Accordingly, determining by ear that sine tones that have a particular acoustical frequency are the same in perceived pitch as non-sine tones goes hand in hand with hearing the sine tones and the non-sine tones as parts of a single group. Conversely, any differences in perceived pitch would be heard as additional differences between the group that consists of the first and third tones and the group that consists of the second and fourth tones.

Returning to Kyai Parijata's saron tones, one finds that the spectra of the saron demung's tones are more complicated than those of the saron barung. As with the saron barung tones, a sine tone whose frequency was the same as the frequency of the saron demung tone's loudest partial was heard as the same in pitch for five of the seven tones. In contrast, two of the saron demung tones were somewhat challenging to match with a sine tone. For these, I had to generate a sine tone whose frequency was a little greater than the loudest frequency of the corresponding saron demung tone.



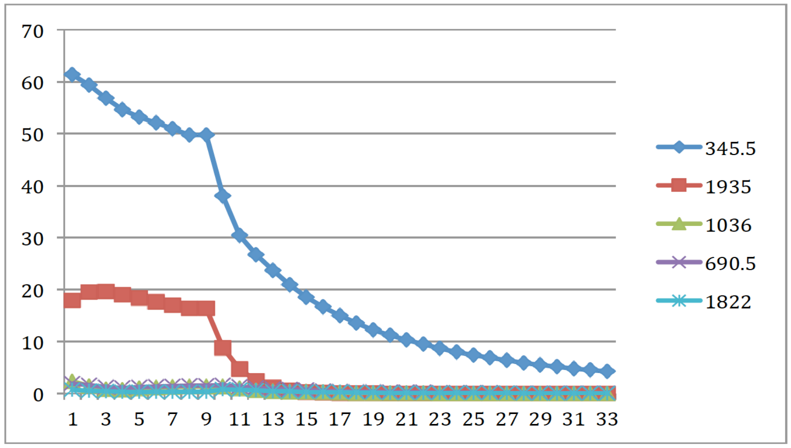
Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 1.



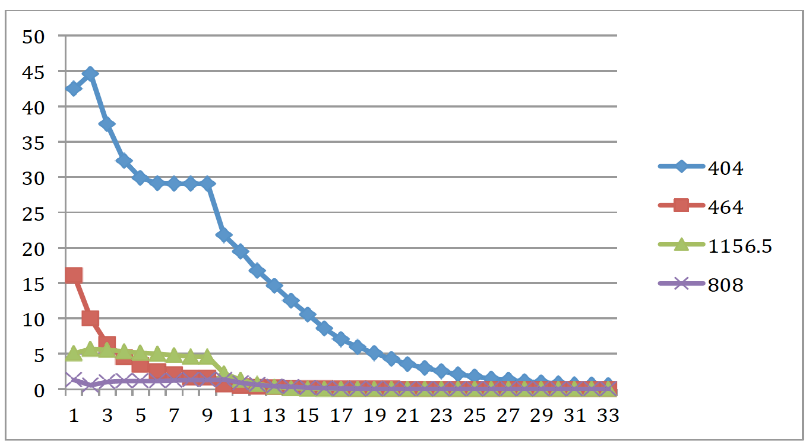
Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 2.



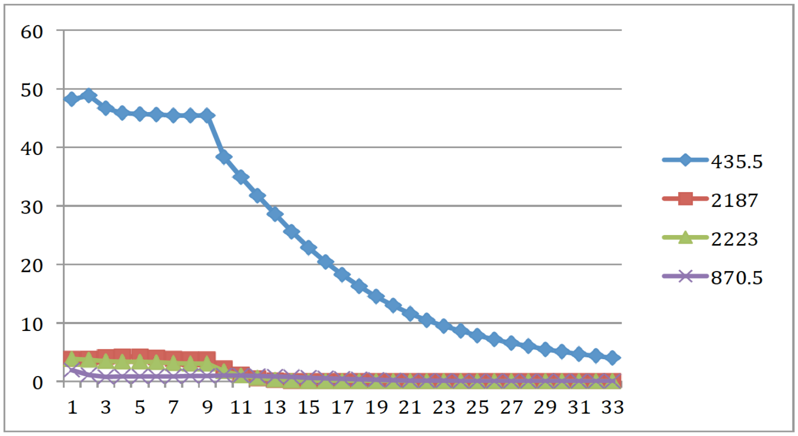
Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 3.



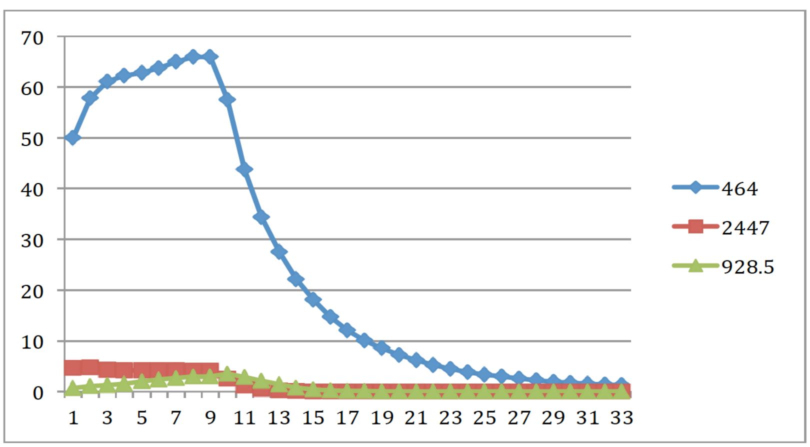
Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 4.



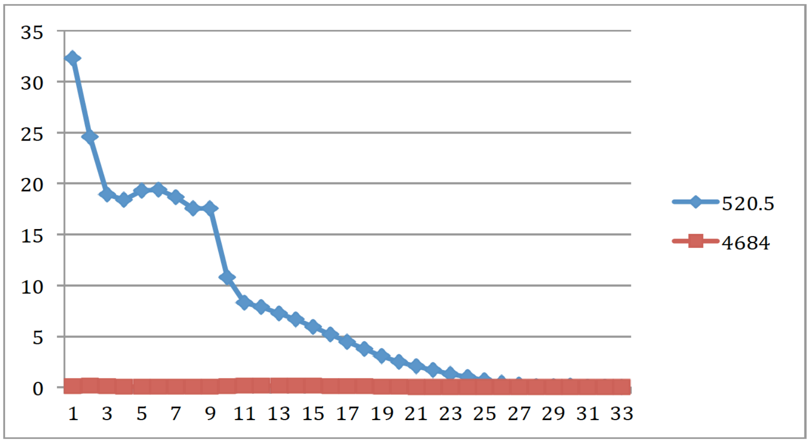
Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 5.



Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 6.



Y-axis: Loudness in phons
X-axis: Time—1 to 8 in 12.5-ms increments, 9 to 33 in 200-ms increments
Colored data entries: Frequency in Hz

Saron demung, key 7.

In listening to both of the adjusted sine tones within a succession of four 500-ms tones, I experienced the following anomaly for particular sine-tone frequencies: if I subjectively accented the second and fourth tones, which were demung tones, I heard them as higher than the first and third tones, which were sines that I had adjusted slightly upward; conversely, if I subjectively accented the first and third tones, I heard them as higher than the second and fourth.

In this audio example, the sine tones have the same frequency as the loudest frequency of demung tone number 1. In this example, the sine tones have a slightly greater frequency than the loudest frequency of the same demung tone.

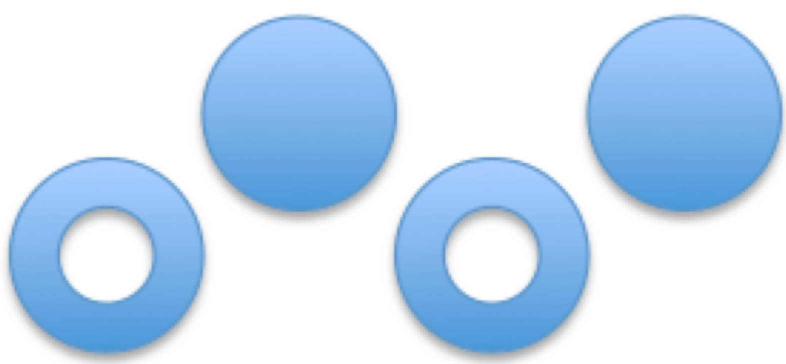
As one finds in experiments designed to determine just-noticeable differences, hearing two tones as matching in pitch is a matter of greater or less probability within a particular range of frequencies. As a consequence, it is not entirely surprising that for particular tones that are not sine tones and that do not have harmonic, overtone-series spectra, certain frequency values will have a relatively high probability of being heard as either higher than or lower than a particular sine tone depending on such an additional factor as subjective accentuation (concerning which, see, e.g., Temperley 1963, 267). Plausibly, then, such subjective acts of metrical auditory cognition are what Schneider was referring to when he characterized saron tones as "uncertain" with regard to pitch.

The next two images provide a crude visual–spatial counterpart to the auditory, perceptual phenomenon.

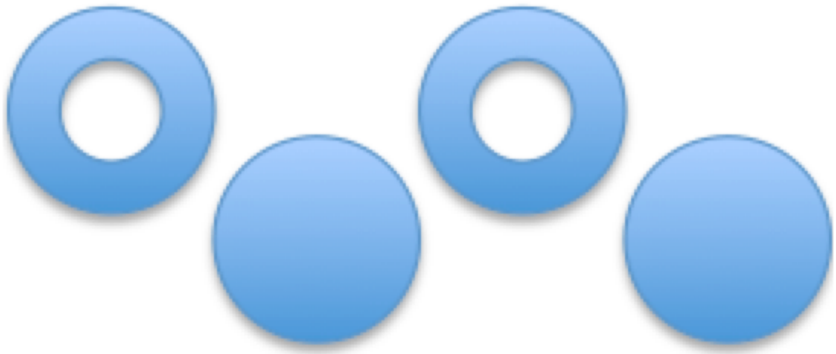
Same four tones with alternating timbres: second and fourth tones heard as higher.

Same four tones with alternating timbres: second and fourth tones heard as lower.

In my own experience, such vacillation occurs only within a relatively small range of frequencies. Subsequently, I clarify what I have just referred to as "a relatively small range of frequencies." In the meantime, one can note that a sine-tone frequency at which subjective accentuation tilts one's perception of pitch in both directions would provide a fair estimate of "the" pitch of such uncertain tones. In the next pair of images, the amount of perceived difference is much greater.

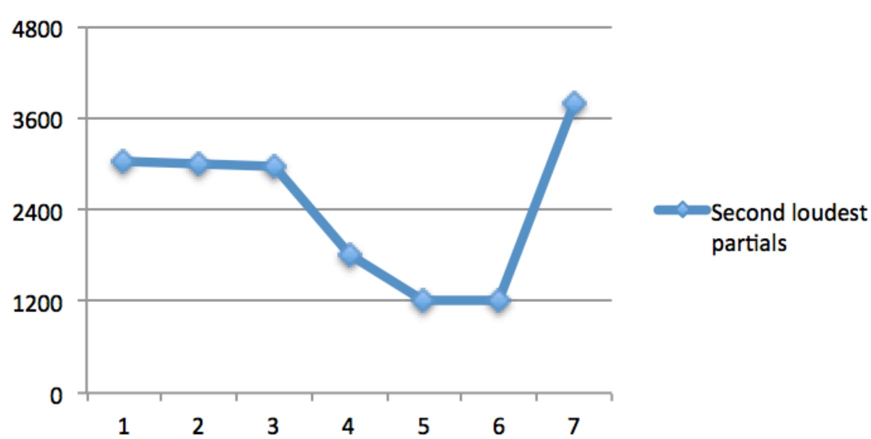


Second and fourth tones heard as much higher.



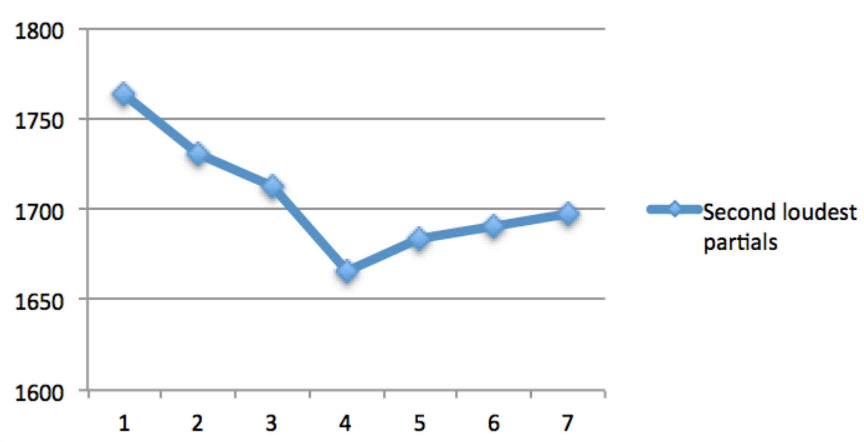
Second and fourth tones heard as much lower.

Within each of the saron demung tones produced by Kyai Parijata in this audio example, there is at least one partial that is both perceptually prominent and audibly higher than the tone's loudest partial, which corresponds to the pitch of the tone as a whole. Although the loudest, most prominent of these upper partials is softer than the fundamental, it is audible, partly because it is not immersed in an overtone series above the fundamental, and partly because it is more than an octave above the fundamental. In this regard, the sound recordist for Kyai Parijata has said, the ensemble as a whole is quite irregular in timbre (Oldenborgh 2002). Whereas Kyai Parijata's saron barung tones are effectively sine tones, the ensemble's saron demung tones comprise prominent upper partials that seem to scintillate above the fundamental, without masking it. Moreover, as the next image shows, the second-loudest partials differ substantially in the amounts by which they are higher than their fundamentals. The y-axis represents cents above fundamental frequencies, while the x-axis represents tones 1 to 7 of the saron demung.

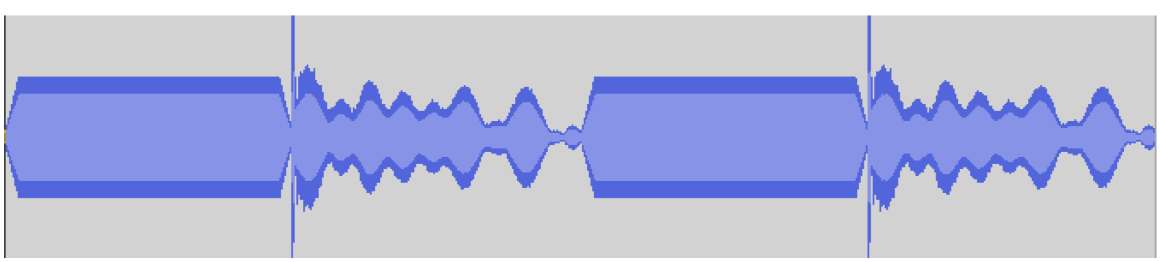


To be sure, all of the second-loudest partials are an octave or more above their fundamentals. However, they range from one to three octaves, i.e., from ca. 1200 cents to ca. 3600 cents. As well, three of them are close to one or three octaves above their fundamentals, whereas four are close to one or two octaves plus a tritone.

To avoid generalizing unduly about sarons, I felt it prudent to compare Kyai Parijata's saron tones with those of another gamelan. In this regard, Bill Sethares kindly sent me .wav files he had recorded of individual saron tones of the gamelan at the home of Central Java's foremost musician of the past century, the late K.P.H. Notoprojo (1909–2007), also known familiarly as Pak Cokro. In contrast to the Kyai Parijata ensemble, the second-loudest partials produced by the saron demungs of the Pak Cokro gamelan are all fairly close to an octave plus a perfect fourth above their fundamentals, i.e., about 1700 cents higher.



The Pak Cokro sarons afforded an opportunity to consider an additional aspect of saron tuning. As mentioned earlier, if there is more than one saron instrument playing one of the two saron parts, all of them play the same numbered key at the same time. In the Pak Cokro ensemble there are three saron barungs. In [this audio example](), you can hear three simultaneous tones alternating with a sine tone that is heard as the same pitch.



Acoustically, none of the loudest partials of each saron barung has the same frequency as either of the others. As a consequence, each three-tone grouping produces beating and/or roughness, which is both acoustically evident in its waveform display above and quite audible. As well, upper partials an octave or more above the fundamentals are heard as a kind of auditory scintillation additional to the loudest partials. As it turns out for all seven saron barung tones, the frequency of the sine tone that matched the pitch of all three was the average of the lowest and highest frequencies, rather than the average of all three as authoritative psychoacoustical accounts might seem to imply (e.g., Gibson, n.d.). Thus:

Tone 1: 607.5 Hz

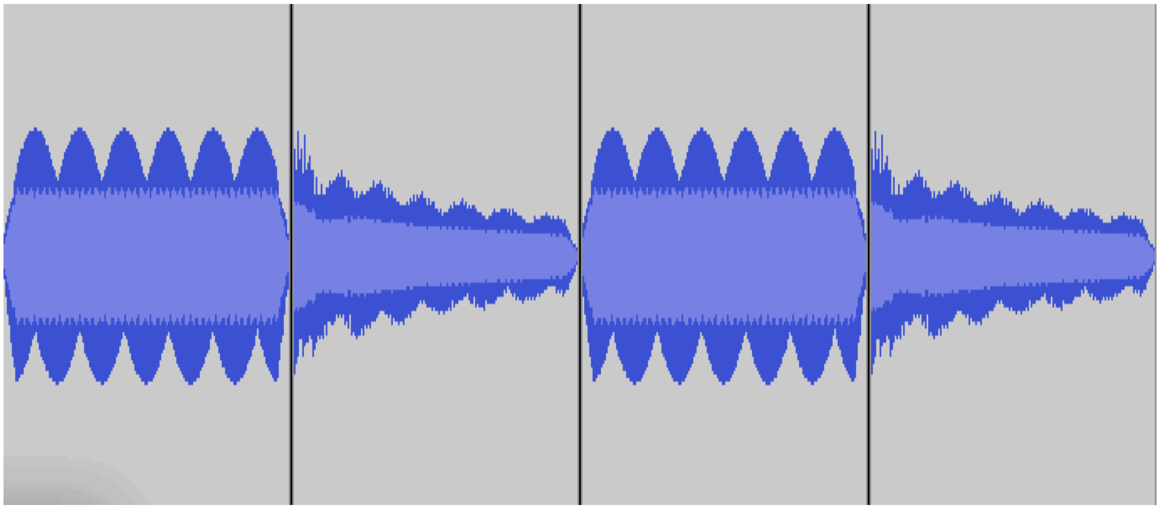
Tone 2: 611.5 Hz

Tone 3: 616.5 Hz

Average: 612 Hz + 9 beats/sec. plus a second loudest frequency of 1597 Hz

This observation is of consequence for formulating intervals and scales, and for analyzing skeletal melodies, because all three tones are heard as a single thing with regard to pitch. Indeed, they are heard as a single thing with regard to timbre in that their beating and/or roughness produces a single pattern of loudness fluctuation along with an upper scintillation.

When the three barung tones and the single demung tone of the Pak Cokro ensemble are played simultaneously, as is usual in performance of a piece's skeletal melody, beats occur among all four tones. Listening to this audio file, one can hear these beats in contrast to the beats of sine tones whose frequencies correspond to them in pitch. As with a group of three barung tones, a group of three barung tones plus a demung tone is heard as a single thing with regard to both pitch and timbre, where timbre includes not only beating and/or roughness, but also scintillation produced by the combined tones. In contrast, the beats produced by Kyai Parijata's demung and barung are quite even and rather subtle in their loudness fluctuations (see image below). In this audio example, key 1 of the saron barung of Kyai Parijata, with its upper partial at 1701.5 Hz, and key 1 of the saron demung are heard in alternation with sine tones having the same fundamental frequencies. All the same, in both ensembles, the frequencies of the two sine tones that match simultaneous demung and barung tones require an even smaller upward adjustment than the two, "uncertain" saron demung tones when they are heard in isolation rather than in combination with other saron tones.



In sum, there is considerable spectral diversity among the saron tones of the two ensembles. Some are effectively sine tones, others have more or less prominent upper partials, that scintillate far above the loudest, fundamental partial, or interact with it by doubling the fundamental one, or even three, octaves above. All but two can be directly matched perceptually in pitch with a sine tone that has the same acoustical frequency as their loudest partial. Those two require a sine tone of slightly higher frequency and are "uncertain" in the sense that their pitch relationship with a sine tone a few cents higher seems to shift from higher to lower depending on one's subjective metrical orientation. In any event, all the saron tones that would be employed in skeletal melodies are tractable with regard to pitch. That is, one can determine that each is heard as matching a particular sine tone when it is heard in isolation, and similarly for tones produced simultaneously by saron keys that have the same number. As a consequence, the acoustical frequency of such sine tones can be considered a fair surrogate estimate for a particular saron tone's pitch.

Scale(s)

The table below lists acoustical frequencies and pitch surrogates for Kyai Parijata's 14 saron tones.

demung tones	1	2	3	4	5	6	7
acoustical	**293.5**	**316**	345.5	407	435.5	464	520.5
simultaneous	***296.75***	***319***	*345.5*	*407*	*435.5*	*464*	*520.5*
isolated	**298.5**	**322**	345.5	407	435.5	464	520.5

barung tones	8	9	10	11	12	13	14
acoustical	595.5	640.5	697	821	883	940	1059.5
simultaneous	*593.5*	*640.5*	*697*	*821*	*883*	*940*	*1058.5*
isolated	593.5	640.5	697	821	883	940	1059.5

The top row shows acoustical frequencies of the loudest partials; the middle row shows pitch surrogates for tones produced simultaneously by demung and saron keys having the same number; the bottom row shows pitch surrogates for tones produced in isolation, rather than simultaneously with the other instrument. Among these, the only differences involve tones produced by keys 1 and 2 of the saron demung, highlighted in boldface. Since the sarons perform skeletal melodies in simultaneous combination, I focus presently on the values in the middle, italicized row of the slide.

So far, I have assumed only that one tone can be heard as higher than, lower than, or the same in pitch as another tone. In proceeding from pitches to intervals to scales, I introduce two more kinds of relationship. The first kind of relationship is defined in terms of the lower-than relationship:

(x)(y)(xHLTy . xHAPy ⟵⟶ xHLTy . –(∃z)(xHLTz . zHLTy))

For any things (e.g., tones), x and y, x is heard as lower than, and pitchwise adjacent to, y

if and only if

x is heard as lower than y,

and there is no tone, z, such that x is heard as lower than z, and z is heard as lower than y.

According to the second kind of relationship, one pair of tones is heard as pitch-intervallically smaller than another pair of tones. A weak postulate that links surrogate pitch values to pairs of tones, and hence, to scales, is the following:

If tone-pair x is heard as pitch-intervallically smaller than tone-pair y, then

the ratio of x's surrogate pitch values is smaller than the ratio of y's surrogate pitch values.

In what follows, I specify the ratio of a pair of tones' surrogate pitch values in terms of cents, i.e., hundredths of a tempered semitone.

The next table lists the pitch-surrogate ratios among pairs of adjacent tones produced by Kyai Parijata's sarons. Each of these ratios corresponds to a step, and is expressed in cents.

demung:

tones	1	2	3	4	5	6	7
Hz	296.75	319	345.5	407	435.5	464	520.5
cents above demung1:	0	125	263	547	664	774	973
cents between steps:	125	138	284	117	110	199	227

barung:

tones	8	9	10	11	12	13	14
Hz	593.5	640.5	697	821	883	940	1058.5
cents above demung1:	1200	1332	1478	1762	1888	1996	2202
cents between steps:	132	146	283	126	108	206	

More than 50 years ago, J. Murray Barbour (1963, 320) suggested that pélog comprises five small (*kleine*) steps and two large (*grosse*) steps. Barbour's binary opposition might seem, at first, somewhat vague: How small is small? How large is large?

In the next table, rectangles surround the ratios for the largest small step and the smallest Large step.

demung:

tones	1	2	3	4	5	6	7
Hz	296.75	319	345.5	407	435.5	464	520.5
cents above demung1:	0	125	263	547	664	774	973
cents between steps:	125 small	138 small	284 Large	117 small	110 small	199 small	[227 Large]

barung:

tones	8	9	10	11	12	13	14
Hz	593.5	640.5	697	821	883	940	1058.5
cents above demung1:	1200	1332	1478	1762	1888	1996	2202
cents between steps:	132 small	146 small	283 Large	126 small	108 small	[206 small]	

Since the difference is only about a fifth of a tempered semitone, one might regard the distinction between Large and small to be somewhat arbitrary. Nonetheless, one can formulate small and Large as relative sizes in a way that is both verifiable and falsifiable, as well as consistent throughout all the tone-pairs among the 14 saron tones. If such relative sizes are combined with the number of steps each pair of tones spans, they provide a basis for understanding an acoustically measured tuning as a perceived scale—more specifically, a perceived scale that constitutes a perceptual whole. The table below shows ordered degree-class intervals of an octave or less among the tones of Kyai Parijata's sarons.

categories:	**sizes (in cents):**															
steps, size	**min**	**max**	**tone-pairs**													
0, 0	**0**	**0**	**1-1**	**2-2**	**3-3**	**4-4**	**5-5**	**6-6**	**7-7**	**8-8**	**9-9**	**10-10**	**11-11**	**12-12**	**13-13**	**14-14**
1, s	108	**206**	1-2	2-3		4-5	5-6	6-7		8-9	9-10		11-12	12-13	13-14	
1, L	**227**	284			3-4				7-8			10-11				
2, 2s	227	**314**	1-3			4-6	5-7			8-10			11-13	12-14		
2, s+L	**359**	430		2-4	3-5			6-8	7-9		9-11	10-12				
3, 3s	426	**440**				4-7							11-14			
3, 2s+L	**506**	562	1-4	2-5	3-6		5-8	6-9	7-10	8-11	9-12	10-13				
4, 3s+L	669	**723**	1-5	2-6	3-7	4-8	5-9	6-10		8-12	9-13	10-14				
4, 2s+2L	**789**	789							7-11							
5, 4s+L	774	**870**	1-6	2-7		4-9	5-10			8-13	9-14					
5, 3s+2L	**915**	988			3-8			6-11	7-12							
6, 5s+L	931	**1002**	1-7			4-10				8-14						
6, 4s+2L	**1023**	1114		2-8	3-9		5-11	6-12	7-13							
7, 5s+2L	**1200**	**1229**	**1-8**	**2-9**	**3-10**	**4-11**	**5-12**	**6-13**	**7-14**							

As this and the following table show, Kyai Parijata's 14 saron tones comprise 23 distinct categories. All the tone-pairs in each category span a particular number of steps and bear a particular size relationship to tone-pairs in the other category that spans the same number of steps. In order to specify the empirical boundaries of these relative sizes, the above table lists each category's smallest and largest tone-pairs as a minimum ratio and a maximum ratio, again expressed in cents. In both tables, the leftmost column lists categories of tone-pairs according to the number of steps they span and by their relative sizes. The next pair of columns lists the minimum and maximum sizes, in cents, for all the tone-pairs in each category. The remaining entries show the numbers of the saron tones, from 1 to 14, that comprise each tone-pair.

As shown in the table above, the ratio of each 1-step tone-pair whose relative size is "small" is smaller than the ratio of all 1-step tone-pairs whose relative size is "Large." Starting from the small and Large values of the one-step tone-pairs, hypotheses about the relative sizes of all the other tone-pairs are derived by adding the hypothetically small and Large steps they span. For instance, the value of the ratio of each two-step tone-pair whose relative size is “small+small” (or for the sake of abbreviation, “2s”) is hypothesized to be smaller than the ratio of each two-step tone-pair whose relative size is “small+Large” (or “Large+small,” both abbreviated as “s+L” in the table). And so forth. To facilitate these comparisons, the maximum size of tone-pairs hypothesized to be smaller is highlighted in boldface, as is the minimum size of tone-pairs hypothesized to be Larger.

As it turns out, all of the resulting hypotheses are confirmed by each of the tone-pairs. With only two exceptions, all the tone-pairs that span a particular number of steps comprise two categories with regard to their relative sizes. These exceptions are the tone-pairs that span seven steps, i.e., the octaves, and those that, as a limiting case, span zero steps, i.e., the primes. The primes, whose relative size is zero, are smaller than all other the tone-pairs. All the tone-pairs in the octave category are larger than all the tone-pairs that span fewer than seven steps, and, as the next table shows, all the octave tone-pairs are smaller than each of the tone-pairs that span more than seven steps. This table shows ordered degree-class intervals of an octave or more among the tones of Kyai Parijata’s sarons.

categories: steps, size	**sizes (in cents): min**	**max**	**tone-pairs**						
7, 5s+2L	**1200**	**1229**	**1-8**	**2-9**	**3-10**	**4-11**	**5-12**	**6-13**	**7-14**
8, 6s+L	1332	**1428**	1-9	2-10		4-12	5-13	6-14	
8, 5s+3L	**1498**	1498			3-11				
9, 7s+2L	1449	**1538**	1-10			4-13	5-14		
9, 6s+3L	**1624**	1637		2-11	3-12				
10, 8s+2L	1655	**1655**				4-14			
10, 7s+3L	**1733**	1763	1-11	2-12	3-13				
11, 8s+3L	1871	1938	1-12	2-13	3-14				
[11, 7s+4L]	n.a.	n.a.							
12, 9s+3L	1996	2076	1-13	2-14					
[12, 8s+4L	n.a.	n.a.]							
13, 10s+3L	2202	2202	1-14						
[13, 9s+4L	n.a.	n.a.]							

As well, these seven-step, octave tone-pairs can be considered perceptual octaves in the sense that the spectra of their lower and upper tones interact with each other. The acoustical result of this interaction is interference. The perceptual result is audible beating and/or salient roughness, as in the audio example above where tone 1 of the saron demung and tone 1 of the saron barung were heard simultaneously.

The preceding observations hold not only for the surrogate values of tone-pairs heard simultaneously, but also for tone-pairs heard in isolation and for the acoustical values of their fundamental frequencies. To be sure, the pitch surrogates for two saron demung tones, namely, those numbered 1 and 2, when heard together with their octaves, are 19 cents and 16 cents higher than the acoustical values of their fundamental frequencies. Moreover, the pitch surrogates for these two tones, when heard in isolation, are an additional 10 and 16 cents higher than their fundamental frequencies. Nonetheless, the hypotheses based on small and Large one-step tone-pairs are confirmed for all three kinds of values.

The latter result is of importance in assessing previously published accounts of pélog tunings. Since the invention of the Stroboscopic tuner 80 years ago (Banks 2010), several studies of pélog tuning have been based on acoustically determined fundamental frequencies of individual tones rather than on the present study's method of conveying perceived pitch-matches with sine tones by means of surrogate pitch values. Relative to the possibility of divergences of as much as 32 cents between the two approaches when assessing pitches in isolation and the even greater relevance to the present study of much smaller divergences with regard to tones sounding simultaneously, there is no prima facie reason to doubt the perceptual relevance of such earlier studies.

Most notable among such studies are the fundamental-frequency values of saron demung and saron barung instruments in 30 outstanding Central Javanese ensembles for which Wasisto Surjodiningrat and his colleagues (1972) published measurements more than 40 years ago. As shown by a recently posted database (Rahn 2016), each of these confirms the small-and-Large hypotheses.

In sum, there are no anomalous values among the 406 saron tones assessed thus far: 14 in Kyai Parijata, 28 in the Pak Cokro ensemble, and 364 in Surjodiningrat's study. Accordingly, one can conclude that the small-and-Large hypotheses, which are falsifiable, have also been verified to a considerable extent.

Of meta-theoretical and meta-analytical interest, the relative sizes of the tone-pairs are, strictly speaking, non-numerical. In contrast to the abstract, numerical values of much music theory since ca. 300 BCE, the smaller-than and larger-than relationships that are a basis for the present tuning analysis obtain between concrete pairs of tones and are not numerical. In any number system, whether it comprises natural numbers, rational numbers, or real numbers, there is an equality relation. In contrast, the present formulation does not specify that tone-pairs within any of its categories (except, of course, the category of primes) are equal in size to each other.

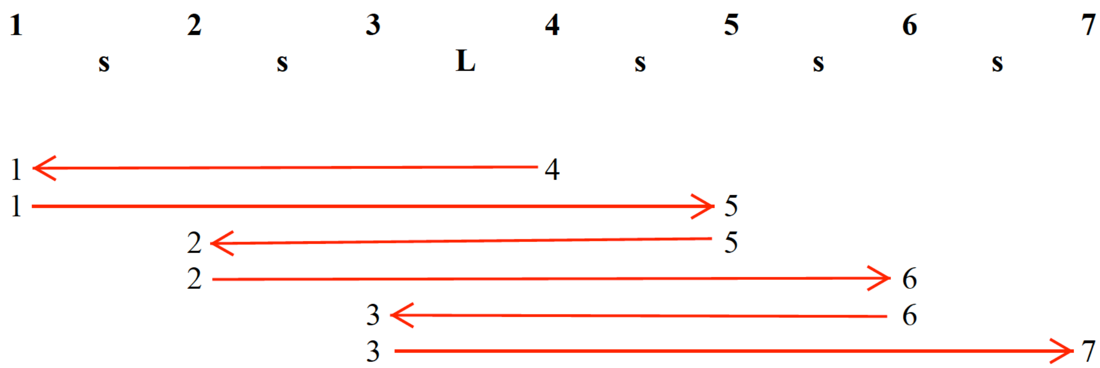
Instead, their status as members of a particular category depends on their being smaller than or larger than tone-pairs in other categories. In mathematical and logical terms, one can say that the tone-pairs in each category are modeled as members of an equivalence class, rather than as numbers (European Mathematical Society 2014). In this sense, the tone-pairs within a category or equivalence class are effectively the same even though their sizes as individual surrogate ratios might not be equal to one another.

Although numbers constitute a usual lexicon for music theory and analysis, and although the sizes of tone-pairs are not formulated here as numbers, aspects of these non-numerical tone-pairs have counterparts in concepts developed in post-War atonal and serial music theory. In this regard, an important difference between the present formulation on one hand, and on the other hand, the mainstream of atonal and serial theory, is that the present formulation assesses intervals in terms of both their relative sizes and the number of steps they span. Nonetheless, counterpart concepts from recent theory are relevant to relationships between tone-pairs within pélog as a scale and are germane to analyzing pélog skeletal melodies.

Tone-pairs 1 and 8, 2 and 9, and so forth can be considered instances of a modular interval that spans seven steps and whose relative size is five smalls plus two Larges, and both tones can be considered instances of the same *degree class* (as a counterpart to a pitch class in atonal and serial theory). Tone-pairs whose constituent steps differ by a whole-number multiple of seven and whose relative sizes differ by a whole-number multiple of five smalls plus two Larges can be considered instances of the same *ordered degree-class interval* (as a counterpart to an ordered pitch-class interval, directed pitch-class interval, or pitch-interval class). For instance, tone-pairs 1 and 4, 8 and 11, and 1 and 11, span, respectively, three, three, and ten steps, and their relative sizes are two smalls plus one Large, two smalls plus one Large, and seven smalls plus three Larges.

Tone-pairs whose constituent steps add up to a whole-number multiple of seven and whose relative sizes add up to a whole-number multiple of five smalls plus two Larges can be considered inversions of each other and are considered instances of the same *step-and-size class* (as a counterpart to an interval class or unordered pitch-class interval or undirected interval). For instance, tone-pairs 1 and 4, and 4 and 8, span, respectively, three and four steps for a total of seven steps, and their relative sizes are two smalls plus one Large and three smalls plus one Large for a total of five smalls plus two Larges.

Concerning the latter point, each of the saron tones in the pélog tunings considered here is part of a cycle that comprises only instances of a single step-and-size class: on one hand, tone-pairs that span three steps and whose relative sizes are two smalls plus one Large; on the other hand, tone-pairs that span four steps and whose relative sizes are three smalls plus one Large.



The numbered saron tones of this cycle are as follows: 4, 1, 5, 2, 6, 3, and 7, or conversely, 7, 3, 6, 2, 5, 1, and 4. Of potential historical or lexical interest, "pélog" is the Indonesian term for tone 4, and this fourth of seven tones constitutes the first or last tone of the cycle. Moreover, as shown by the preceding two tables and the data set mentioned above (Rahn 2016), none of the largest three-step intervals is larger than any of the smallest four-step intervals, and similarly for the largest ten-step and smallest eleven-step intervals.

A couple of consequences of this cyclic structure are of considerable analytical importance. To employ the term Norman Carey and David Clampitt (1989) proposed more than 20 years ago, such a cyclic structure is a necessary and sufficient condition for a group of tones to be considered "well formed" (Carey 1998). Other well-formed scales include the diatonic collection and the scale John Clough and Jack Douthett (1991, 123) termed the "usual pentatonic," an instance of which can be represented by the letter-names CDEGA.

Among the features of all well-formed scales, one is of relevance to an expanded notion of the Gestalt Principle of Similarity. Specifically, among all possible scales whose modulus spans a particular number of steps (e.g., in this case, a seven-step octave) and whose one-step intervals have more than one size (e.g., in this case, the relative sizes "small" and "Large"), a well-formed scale maximizes the number of tone-pairs that are "the same" with regard to both the number of steps they span and their relative size.

In the pélog tunings considered here, there are seven step-and-size classes within an octave.

Step-and-size classes:			**Number of step-and-size class intervals**	**Number of pairs of step-and-size class class intervals (cf. 2*C(n,2) = (n)(n-1)/2)**	
steps, size		**steps, size**			
0, 0	or	7, 5s+2L	7	2*(7)(6)/2 =	42
1, s	or	6, 4s+L	5	2*(5)(4)/2 =	20
1, L	or	6, 5s+2L	2	2*(2)(1)/2 =	2
2, 2s	or	5, 3s+2L	3	2*(3)(2)/2 =	6
2, s+L	or	5, 4s+L	4	2*(4)(3)/2 =	12
3, 3s	or	4, 2s+2L	1	2*(1)(0)/2 =	0
3, 2s+L	or	4, 3s+L	6	2*(6)(5)/2 =	30
total number of pairs of step-and-size class intervals (cf. 2*(s+1)(s)(s-1)/6):					**112**

Tone-pairs in one of the step-and-size classes, namely the octave and prime, span zero or seven steps and their relative sizes are, respectively, zero, or five smalls and two Larges. Within this class, there are seven instances. Expanding the usual notion of an interval vector in atonal theory to include not only the size of intervals in a particular category but also the number of steps they span, one can include in the "vector" of a pélog scale "7" as the number of seven-step, five-small-plus-two-Large intervals in the octave or prime category. Tone-pairs of another step-and-size class span one step or six steps, namely the second and seventh, and their relative sizes are one small, or four smalls and two Larges. Within this class, there are five instances, each of which might be considered a "minor second," i.e., one of the "smaller" seconds. For these intervals, the vector entry is "5." Tone-pairs in yet another step-and-size class span one step or six steps and their relative sizes are one Large or five smalls and one Large. Within this class, there are two instances, both of which can be considered "major seconds." For these intervals, the vector entry is "2." And so forth.

The number of pairs of ordered degree-class intervals within each of these step-and-size classes is the number of combinations of the ordered degree-class intervals taken two at a time. In the present instance, the number of combinations of the seven ordered degree-class intervals that correspond to an octave or prime and that are taken two at a time is C(7,2) = (7)(7-1)/2 = (7)(6)/2 = 21. Similarly, the number of combinations of the five ordered degree-class intervals that correspond to "minor seconds" and "major sevenths" and that are taken two at a time is C(5,2) = (5)(5-1)/2 = (5)(4)/2 = 10. And so forth.

If s is the number of steps that the modular interval spans, the sum of these numbers of *pairs* of step-and-size intervals is necessarily two times the tetrahedral (or triangular pyramidal) number (s+1)(s)(s-1)/6: in this case, (2)(7+1)(7)(7-1)/6 = (8)(7)(6)/3 = 112.[3]

Among all possible collections of seven step classes in which at least one instance of a step class differs in size from another instance of the same class, such a well-formed collection as pélog comprises the greatest number of pairs of step-and-size intervals, namely, (s+1)(s)(s-1)/3. Like the diatonic collection, pélog maximizes Similarity in comparison with any other scale that comprises seven steps and among whose intervals that span the same number of steps at least two intervals differ in size, i.e., in comparison with any other "non-degenerate" scale.

Another feature of well-formed scales is of consequence for analyzing skeletal melodies performed by saron demungs and saron barungs. Because all and only the tones in a well-formed seven-step scale like pélog are parts of a single step-and-size class cycle, there are necessarily three sub-cycles within this cycle, and the tones within each of these three sub-cycles constitute a well-formed scale that spans two steps fewer than the cycle as a whole.

7-step pélog

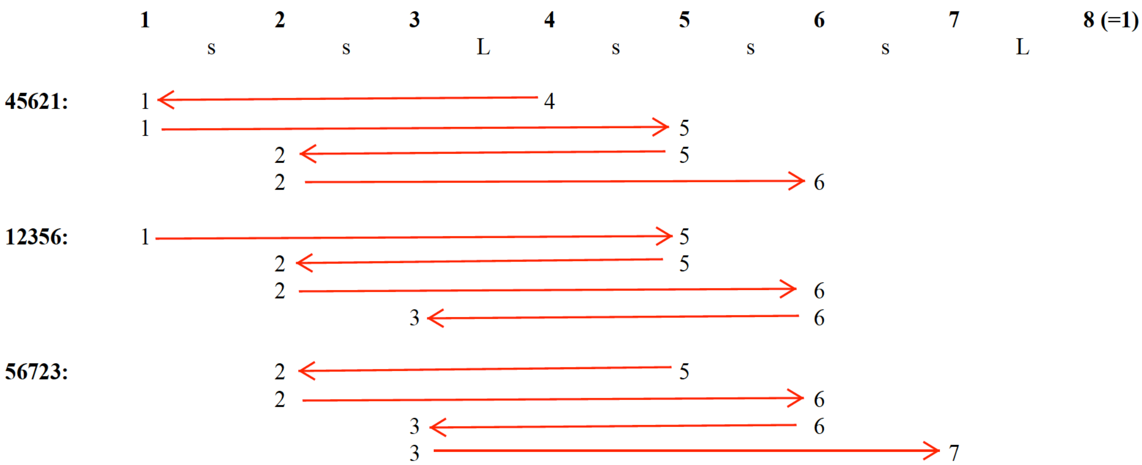
	1		**2**		**3**		**4**		**5**		**6**		**7**		**8 (=1)**		**9 (=2)**
		s		**s**		**L**		**s**		**s**		**s**		**L**		**s**	

3 well-formed 5-step pélog scales

a)			**2**		**3**				**5**		**6**		**7**				**9 (=2)**
				s			s+L			s		s			s+L		
b)	**1**		**2**		**3**				**5**		**6**				**8 (=1)**		
		s		s			s+L			s			s+L				
c)	**1**		**2**				**4**		**5**		**6**				**8 (=1)**		
		s			s+L			s		s			s+L				

[3] Cf. Sloane (1973, A000292); conversely, Rahn (1991) and Carey (2002) base assessments of Similarity (or "sameness") on pairs of intervals that span the same number of steps but differ in size: these comprise the complement of those that are the same in size.

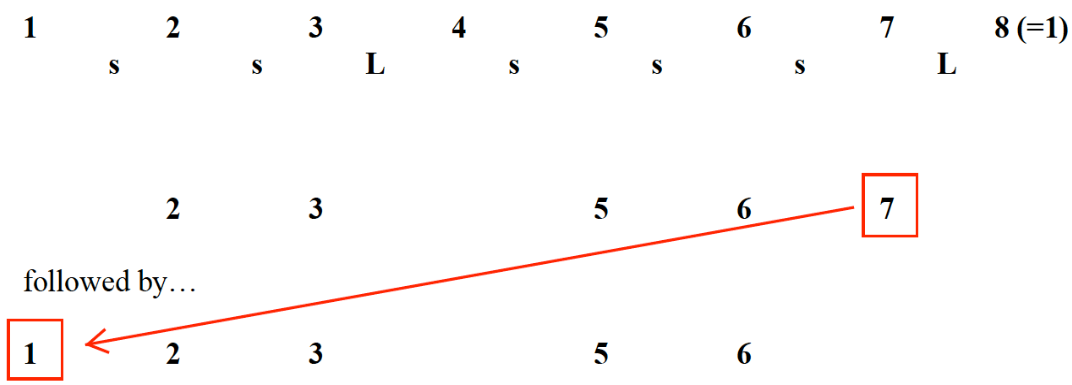
In pélog, each of these three subcycles comprises five steps, and each tone-pair of the step-and-size classes that constitute the generator of each five-step cycle spans two or three steps and, as with the seven-step cycle considered earlier, the generator's relative size is, respectively, two smalls plus one Large, or three smalls plus one Large.



Like the larger cycle of which they are parts, each of the five-step subcycles maximizes step-and-size class Similarity in comparison with any other five-step scale that comprises at least two intervals that span the same number of steps but differ in size. Specifically, the number of pairs of tones that belong to the same step-and-size classes is, again, necessarily (s+1)(s)(s-1)/3: in this case, (5+1)(5)(5-1)/3 = 40.

Step-and-size classes:			**Number of step-and-size class intervals**	**Number of pairs of step-and-size class intervals (cf. 2*C(n,2) = (n)(n-1)/2)**	
steps, size		**steps, size**			
0, 0	or	5, 5s+2L	5	2*(5)(4)/2 =	20
1, s	or	4, 4s+2L	3	2*(3)(2)/2 =	6
1, s+L	or	4, 4s+L	2	2*(2)(1)/2 =	2
2, 2s	or	3, 3s+2L	1	2*(1)(0)/2 =	0
2, 2s+L	or	3, 3s+L	4	2*(4)(3)/2 =	12
total number of pairs of step-and-size class intervals (cf. 2*(s+1)(s)(s-1)/6):					**40**

That such five-step subcycles are relevant to skeletal melodies in pélog tuning is manifest empirically in several ways. For instance, five is the smallest number of degree classes that are employed in any of the 402 pélog skeletal melodies in Barry Drummond's (n.d.) invaluable online compilation of notations for multi-section Central Javanese pieces (*gendhings*). Each of these five-degree-class pieces constitutes one of the well-formed five-step subcycles: specifically, tones 1, 2, 3, 5, and 6, or tones 2, 3, 5, 6, and 7.



Many of the other melodies in Drummond's compilation employ six of the seven pitch classes available in pélog: in particular, 1, 2, 3, 4, 5, and 6; and 1, 2, 3, 5, 6, and 7. Are these to be understood as constituting six-step scales or two overlapping five-step scales? If one counts pairs of ordered degree-class intervals within an octave as before, it turns out that interpreting such six-tone collections as two, overlapping, well-formed five-step scales results in 70 Similarity pairs (i.e., without double-counting). In contrast, interpreting such six-tone collections as a single "unwell-formed" six-step scale results in only 54 Similarity pairs. If maximizing Similarity privileges one interpretation in comparison with another, one can conclude that such six-tone collections are better understood as two overlapping, well-formed five-step scales rather than as a single six-step scale.

In this regard, an important aspect of the skeletal melodies that comprise more than five pitch classes is that the pitch classes that distinguish one five-step subcycle from another are separated temporally. For instance, when a melodic passage that employs 2, 3, 5, 6, and 7 is followed by tone 1, tone 7 is not heard immediately before tone 1. 2, 3, 5, 6, and 7 constitute a well-formed five-step scale, as do 1, 2, 3, 5, and 6. Not hearing 7 and 1 in immediate succession results in the two scales being separated in time to a greater extent than they would be otherwise: an instance of the Gestalt Grouping Principle of Proximity (Wertheimer 1923), or its opposite, Distance, which further groups tones that are, in any event, grouped by Similarity.

In usual analyses of Central Javanese music, such tones as 7 and 1 would be termed *sorogan*, for which "exchange tones," "shifting tones," "alternate tones," "substitute tones," and "auxiliary tones" have been English translations (e.g., Kunst 1973, 39, 49, 94). The ideas here are that 7 and 1 are exchanged, just as one might exchange a quarter for two dimes and a nickel, suggesting parity, or that one of the two tones is somehow subsidiary to the other. As well, the shift from 23567 to 12356 could be termed, following Constantin Brăiloiu's (1955) coinage more than 60 years, ago, a *métabole*.

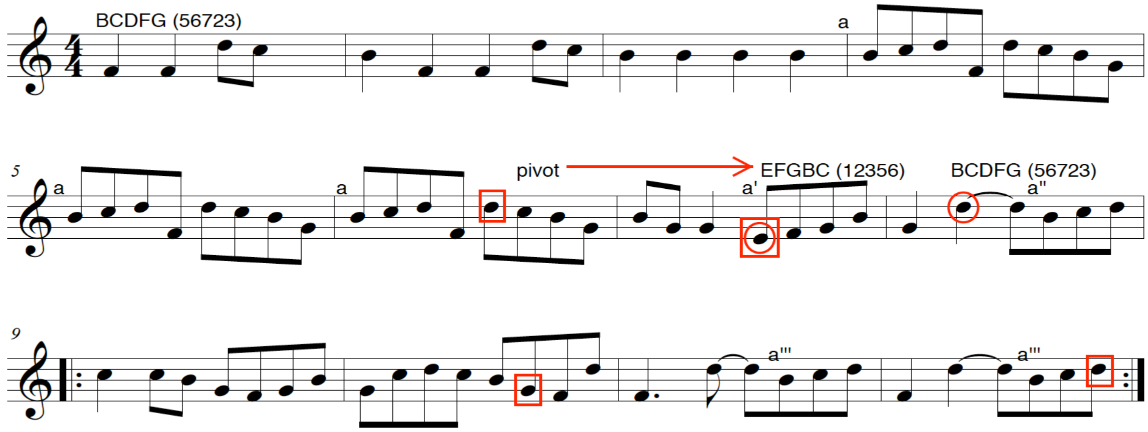
In any event, between the last occurrence of tone 7 and the first occurrence of tone 1, one hears only one or more tones that are shared by both scales. In post-War music theory, these would be termed "common tones," a source of maximal degree-class Similarity throughout a passage that comprises both scales (not to be confused with the narrower notion of "common tones under transposition," for which see, e.g., Morris 2007, 84–85). To borrow a notion from European common-practice theory, one could also regard them as "pivot tones" between two scales, as counterparts to pivot chords shared by two keys.

One might also consider tone 7 to be a "replacement" for tone 1, just as, for example, B♭ could be considered to replace B♮ when a passage comprising the pitches of a C-major scale is followed by the pitches of an F-major scale, or, as another analogy, if F follows a passage comprising CDEGA of C pentatonic, resulting in FGACD of F pentatonic.

Whatever might be one's hermeneutic gloss in such a situation, the introduction of tone 1 after a passage that comprises 23567 is clearly audible as a prominent source of difference that is redressed, or compensated for, by Similarity within each five-degree scale and by the common tones that both scales share, as at the beginning of the skeletal melody for the multi-section piece "Tukung," the opening measures of which are transnotated below.

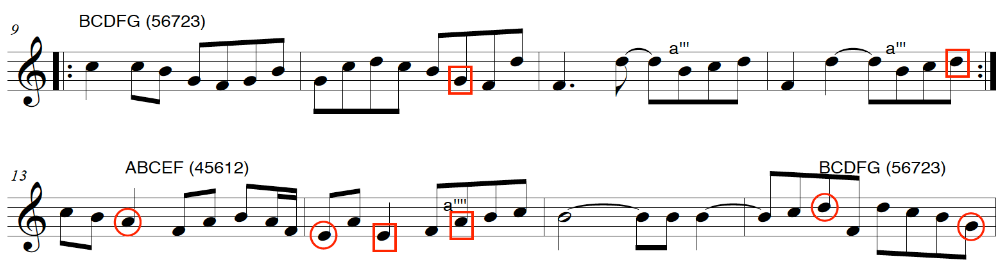
Pieces

To facilitate comprehension, Barry Drummond's (n.d.) cipher notation for "Tukung" has been re-written in European-derived staff notation. In the European-derived staff notation, notes E, F, G, A, B, C, and D correspond to tones 1 to 7, respectively. Accordingly, the initial passage comprises F, G, B, C, and D, which correspond to tones 2, 3, 5, 6, and 7. A square highlights the last instance of D, which distinguishes FGBCD from EFGBC. Tone 1, notated as E and highlighted by a circle, marks the beginning of the five-step scale 12356, which is notated as EFGBC (the preceding C has not been replaced). A later occurrence of tone 7, notated as D, replaces the preceding E and marks the resumption of the five-step scale 23567.

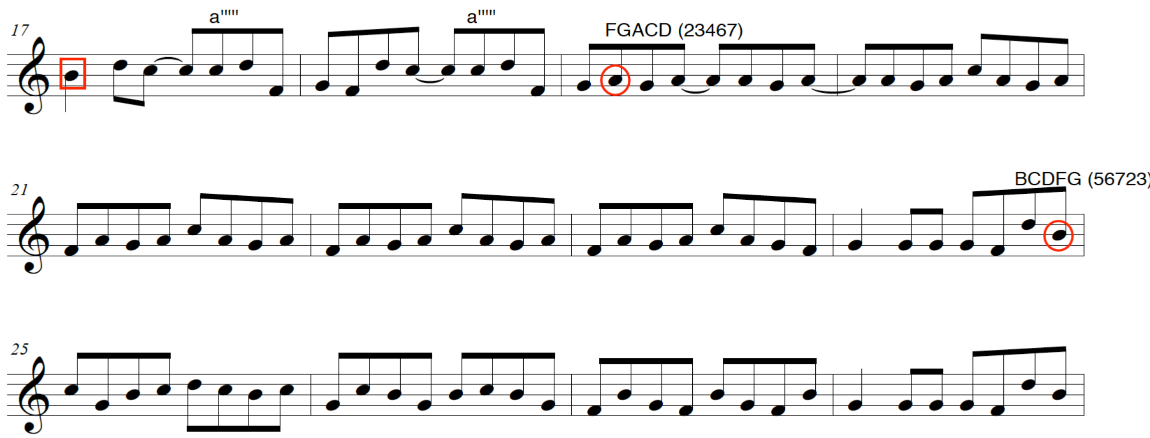


The notation also emphasizes the common tones or pivot tones that exclude both tone 7 and tone 1 between explicit instances of the five-step scales. As well, the notation shows how, as a feature of design, the change from one 5-step scale to another is preceded by motivic repetition, indicated by means of lower-case letters, and notably, the 56727/BCDFD figure recurs as 12353/EFGBG at the introduction of the EFGBC scale.

The second section of "Tukung" begins where the previous part ended, namely, in the five-step scale 23567, and thereupon proceeds to the five-step scale 12456. In this instance, 1 replaces 7 and 4 replaces 3, so that there are only three common tones: 2, 5, and 6. As a consequence, the succession from the 23567 scale to the 12456 scale is more salient by virtue of their sharing fewer tones. In this way, one can acknowledge degrees of Similarity in passages that comprise two scales, just as in European-derived common-practice theory D major is "more remote" from C major than is G major.



Finally, the continuation of the second section of "Tukung" illustrates a different kind of contrast between five-step scales. Whereas the passage begins in the well-formed five-step scale 23567, it does not proceed to another well-formed five-step scale; instead it proceeds to the unwell-formed five-step scale 23467, conveyed in the transcription by the notes F, G, A, C, and D, where tone 5 is replaced by tone 4.



2		**3**		**4**		**6**		**7**		**(9)**
	s		**L**		**2s**		**s**		**L+s**	

steps, size		**steps, size**	**number of interval classes**	**number of pairs of interval classes**	
0, 0	or	5, 5s+2L	5	2*(5*4/2) =	20
1, s	or	4, 4s+2L	2	2*(2*1/2) =	2
1, L	or	4, 5s+L	1	2*(1*0/2) =	0
1, 2s	or	4, 3s+2L	1	2*(1*0/2) =	0
1, L+s	or	4, 4s+L	1	2*(1*0/2) =	0
2, s+L	or	3, 4s+L	1	2*(1*0/2) =	0
2, 3s	or	3, 2s+L	1	2*(1*0/2) =	0
2, 2s+L	or	3, 3s+L	3	2*(3*2/2) =	6
total number of pairs of interval classes:					**28**

The table above shows the number of pairs of interval classes in 23467. On one hand, these five-step scales share four tones, namely, 2, 3, 5, and 6, which are notated as F, G, C, and D. On the other hand, the relative sizes of the one-step tone-pairs in the 23467 scale are much more diverse than their counterparts in 23567: specifically, small, Large, 2 small, small, and small plus Large. Rather than resulting in the 40 interval-class pairs of a well-formed five-step scale, the 23467 scale results in only 28.

As just illustrated, Similarity is sustained in various ways when one five-step scale follows another. Conversely, there are various ways in which contrast is introduced. Nonetheless, only 7 of the 21 possible five-step scales comprise the pattern of 1, 1, 2, 1, and 2 steps in the 7-step scale. Of these 7, only the 4 scales just illustrated are employed among all 402 multi-section skeletal melodies in Drummond's compilation (see the table below, showing the number of pieces within which particular scales are employed). And although "Tukung" combines all 4 of these 5-step scales, only 8 of the 15 possible combinations of the 4 scales are actually employed in any of the 402 pieces.

1	**2**	**4**	**5**	**12**	**14**	**15**	**24**	**25**	**45**	**124**	**125**	**145**	**245**	**1245**
24	0	0	85	0	96	12	0	49	0	0	17	117	0	2
Only 1 or 5 occurs throughout an entire gendhing.				4 occurs only with 1; 2 occurs only with 5; 14, 15, and 25 share 4 of 5 tones.						4 occurs only with 1; 2 occurs only with 5; 14, 15, and 25 share 4 of 5 tones.				4 occurs only with 1; 2 occurs only with 5; 14, 15, and 25 share 4 of 5 tones.

All of the skeletal melodies comprise at least five numbered keys on each saron. The only melodies that comprise only five keys consist of keys 1, 2, 3, 5, and 6, for which I employ "1" as an abbreviation, or keys 2, 3, 5, 6, and 7, for which I employ "5." The scale comprising 12456, which I call "4," is employed only with "1," i.e., with 12356. And the collection consisting of 23467, which I call "2," is employed only with "5."

In these combinations, one can discern a basis for Central Javanese classification of pélog pieces. *Pathet bem*, which in Surakarta comprises *pathet lima* and *pathet nem*, features scale 1, one or more of whose tones (1, 2, 3, 5, and 6) might be employed simultaneously with the final gong of a gendhing. Within individual pathet bem pieces, these tones might be combined with those of scale 5 and/or scale 4. If 1 is combined with both 5 and 4, it might be combined additionally with 2, as in the gendhing "Tamènggita." *Pathet barang* features scale 5, one or more of whose tones (2, 3, 5, 6, and 7) might be employed simultaneously with the final gong of a gendhing. Within individual pathet barang pieces, these tones might be combined with those of scale 1 and/or scale 2. If 5 is combined with both 1 and 2, it might be combined additionally with 4, as in "Tukung."

Conclusion

To conclude, I have shown how one can proceed in "bottom-up" fashion from individual saron tones through a formulation of pélog as a scale or group of scales to an analysis of the skeletal melody in an individual piece and to a scale-based analysis of the skeletal melodies in an entire repertoire. All the saron tones were tractable with regard to pitch: specifically, each tone was heard as matching in pitch a single sine tone, and this sine tone's frequency could reasonably serve as a surrogate value for the tone's pitch, whether the tone was heard in isolation or in combination with another tone with which it was heard as constituting an interval. Further, all the tone-pairs produced by the sarons could be grouped into discrete interval categories on the basis of the number of steps they spanned and their size relative to other tone-pairs that spanned the same number of steps. The resulting intervals constituted a single well-formed seven-step scale and three well-formed five-step scales. And finally, in combination with a single unwell-formed five-step scale, these well-formed scales audibly informed analysis of a skeletal melody that sarons have performed as well as a large corpus of such melodies.

REFERENCES

Audacity Team. 2008–16. *Audacity*. http://www.audacityteam.org/.

Banks, Margaret Downie. 2010. "Dynamic Research: Earle L. Kent and Conn's Research Department." *National Music Museum Newsletter* 37(2); accessible online at http://collections.nmmusd.org/News/Newsletter/August2010/ConnResearch.html.

Barbour, James Murray. 1963. "Mißverständnisse über die Stimmung des javanischen Gamelans." *Die Musikforschung* 16(4): 315–23.

Brailoiu, Constantin. 1955. "Un problème de tonalité (la métabole pentatonique)." *Mélanges d'histoire et d'esthétique musicales offerts à Paul-Marie Masson*, vol. 1, 63–75. Paris: Richard Masse.

Carey, Norman. 1998. "Distribution Modulo 1 and Musical Scales." Ph.D. dissertation, University of Rochester; accessible online at https://www.academia.edu/3385414/Distribution_modulo_one_and_musical_scales.

———. 2002. "On Coherence, Sameness, and the Evaluation of Scale Candidacy Claims." *Journal of Music Theory* 46(1–2): 1–56; accessible online at https://www.academia.edu/11866001/On_Coherence_and_Sameness_and_the_Evaluation_of_Scale_Candidacy_Claims.

Carey, Norman, and David Clampitt. 1989. "Aspects of Well-Formed Scales." *Music Theory Spectrum* 11: 187–206; accessible online at https://www.academia.edu/3385411/Aspects_of_well-formed_scales.

Clough, John, and Jack Douthett. 1991. "Maximally Even Sets." *Journal of Music Theory* 35: 93–173; accessible online at http://ehess.modelisationsavoirs.fr/atiam/biblio/Clough&DouthettJMT-1991.pdf.

Drummond, Barry. n.d. *Gendhing Jawa: Javanese Gamelan Notation*. http://www.gamelanbvg.com/gendhing/gendhing.html, accessed between 7 January and 4 March, 2016.

European Mathematical Society. 2014. "Equivalence Relation." *Encyclopedia of Mathematics*. https://www.encyclopediaofmath.org/index.php/Equivalence.

Gibson, George N. n.d. "Interference in Time: Beats." *Physics 1075Q: The Physics of Music*. http://www.phys.uconn.edu/~gibson/Notes/Section5_5/Sec5_5.htm.

Hartmann, William M. 1978. "The Effect of Amplitude Envelope on the Pitch of Sine Wave Tones." *Journal of the Acoustical Society of America* 63(4): 1105–13; accessible online at https://www.pa.msu.edu/acoustics/amplenv.pdf.

———. 1997. *Signals, Sound, and Sensation.* Woodbury, NY: American Institute of Physics.

Heins, Ernst L. 1968–69. "Tempo (Irama) in de M.-Javaanse gamelanmuziek: ter introductie van Kjai Paridjata." *Kultuurpatronen (Patterns of Culture)* 10–11: 30–57 (incl. English summary).

Hibbert, Bill. 2011. *The Sound of Bells*. http://www.hibberts.co.uk/wavanal.htm.

Kunst, Jaap. 1973. *Music in Java: Its History, Its Theory and Its Technique.* 3rd ed. The Hague: Martinus Nijhoff.

Morris, Robert. 2007. "Mathematics and the Twelve-Tone System: Past, Present, and Future." *Perspectives of New Music* 45/2: 76-107; accessible online at https://disciplinas.stoa.usp.br/pluginfile.php/229011/mod_resource/content/1/Mathematics%20and%20the%20Twelve-Tone%20System%20%28Morris%202007%29.pdf.

Oldenborgh, Geert Jan van. 2002. Full Resolution Samples of Gamelan Kyai Parijata. https://web.archive.org/web/20151031072118/http://www.marsudiraras.org/gamelan/wav/. Copies of saved .wav files available from Jay Rahn.

Rahn, Jay. 1991. "Coordination of Interval Sizes in Seven-Tone Collections." *Journal of Music Theory* 35(1–2): 33–60; accessible online at https://www.researchgate.net/publication/249882080_Coordination_of_Interval_Sizes_in_Seven-Tone_Collections.

———. 2016. "Small and Large Intervals Produced by *Pélog* Sarons of Central Java." Datatset, accessible online at http://yorkspace.library.yorku.ca/xmlui/handle/10315/32504 and http://yorkspace.library.yorku.ca/xmlui/handle/10315/32503.

Savage, William R., Edward L. Kottick, and Sue Carol DeVale. 1979. "Vibrational Characteristics of Saron Barong Metallophones in the 1893 Field Museum Gamelan." *Journal of the Acoustical Society of America* 66(Supplement 1): S18.

Schneider, Albrecht. 1991. "Psychological Theory and Comparative Musicology." In *Comparative Musicology and Anthropology of Music: Essays on the History of Ethnomusicology,* edited by Bruno Nettl and Philip V. Bohlman, 293–317. Chicago: University of Chicago Press.

———. 2001. "Sound, Pitch, and Scale: From 'Tone Measurements' to Sonological Analysis in Ethnomusicology." *Ethnomusicology* 45(3): 489–519.

Sethares, William A. 2016. Personal communication with the author, 20 January.

Sloane, Neil J. A. 1973. *Handbook of Integer Sequences.* New York: Academic Press; accessible online as *Online Encyclopedia of Integer Sequences (OEIS*) at https://oeis.org.

Surjodiningrat, Wasisto, P. J. Sudarjana, and Adhi Susanto. 1972. *Tone Measurements of Outstanding Javanese Gamelans in Jogjakarta and Surakarta.* 2nd ed. Jogjakarta: Gadjah Mada University Press.

Temperley, Nicholas M. 1963. "Personal Tempo and Subjective Accentuation." *Journal of General Psychology* 68(2): 267–87.

Timer4web. 2016. Changes of www.marsudiraras.org; accessible online at http://www.timer4web.com/domain/www.marsudiraras.org.

Wertheimer, Max. 1923. "Untersuchungen zur Lehre von der Gestalt, II." *Psychologische Forschung* 4: 301–50; translated by Willis Davis Ellis in *A Source Book of Gestalt Psychology*, 71–88, London: Routledge & Kegan Paul, 1938*;* accessible online as "Laws of Organization in Perceptual Forms," in Christopher Green, comp., *Classics*

in the History of Psychology at http://psychclassics.yorku.ca/Wertheimer/Forms/forms.htm.