Quantifying Musical Meter: How Similar are African and Western Rhythm?

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Much has been written during the past century about the similarities and differences between African and Western music (Ward 1927; Waterman 1948; Koetting 1970; Eerola, Himberg, Toiviainen, and Louhivuori 2006). "What makes African rhythms sound so different from Western rhythms?" is a question often asked (Tan, Pfordresher, and Harré 2010, 292). Some authors claim that African music has more complex rhythms (Munyaradzi and Zimidzi 2012), or that its rhythms are more developed (Muchimba 2008, 113). The rhythms in African music have also been compared to those found in Indian music in terms of complexity (Thul and Toussaint 2008a) and their additive/divisive properties (Entsua-Mensah 2015). Comparing Western (European) music to both African and Indian music, Benjamin I. Gilman (1909, 534) writes: "Hindu and African music is notably distinguished from our own by the greater complication of its rhythms. This often defies notation." Kofi Agawu chronicles a good deal of the literature that focuses on the purported prominence of rhythm in African music, and its asserted complexity relative to that of Western music (Agawu 1995). But is it indeed the case that African and Western musical rhythms are fundamentally different? In support of this view John Miller Chernoff (1979, 54) writes that "Western and African orientations to rhythm are almost opposite." On the other hand, for David Temperley (2004, 289) "African and Western rhythms are profoundly similar." What is one to make of such antithetical pronouncements? The present study will not take sides on this dichotomous characterization of a complicated issue. One reason for taking this stance is that there exist in the literature scores of different definitions of rhythm (Toussaint 2013). More than forty years ago Kolinski (1973, 494) reported that the music literature already contained at least fifty definitions of rhythm. By which definition then should African and Western rhythm be compared? As an example, consider one of these definitions penned by B. C. Wade (2004, 57), which stipulates that "A rhythm is a specific succession of durations." By this definition African and Western rhythms are more than "profoundly similar." They both consist of successions of durations. Furthermore, although this study is quantitative in nature, its goal is not to pin down a number with which to characterize the degree of rhythmic similarity lying somewhere in between "almost opposite" and "profoundly similar." A possible way to measure rhythm similarity quantitatively is to calculate a comprehensive list of rhythmic features from both symbolic and acoustic samples of African and Western music, thus rendering the samples as points in a high-dimensional space, in which each dimension represents one feature value. The separation of these points measured according to a suitable metric might then yield a quantitative measure of the similarity of African and Western rhythm. Such an ambitious and difficult study is left for the future. The much more modest goals of the present study are two-fold. The first is a restriction to zoom in on a single property of rhythm, namely hierarchical meter, and to suggest a method of quantifying it by means of pulse saliency histograms. The

second limitation is to calculate these histograms for some specific examples of African and Western music to determine how they can inform the issue of whether African rhythm exhibits hierarchical meter (in the GTTM sense). It is hoped that this approach provides new quantitative data that will illuminate the more general question of the similarity between African and Western rhythm.

This study is exploratory in nature, and a word is in order concerning the samples of Western and African rhythms used, as well as the experimental methodology adopted. The word "African" here refers to the music indigenous in the region to the south of the Sahara desert, and thus excludes the Arabic rhythms of North Africa. However, it includes rhythms from the Caribbean and Brazil that are used by the communities of African ancestry. On the methodological side, the statistical measures employed are descriptive rather than hypothetical. No hypotheses are posited here with regard to whether African and Western rhythm are similar or not. To carry out such a scientific study properly, random samples of all Western and all African music would have to be obtained, in order to be able to validly test hypotheses. The study here is rather a preliminary exploration (in the style of data mining) of some examples that may point the way to more exhaustive analyses and more rigorous tests of specific hypotheses. In the absence of a random sample of African rhythms, an alternative approach is adopted in the form of a worst-case analysis, in which rather than obtaining a large *random* sample of African rhythms, a collection of *unique* special rhythms is selected for study. In particular, if the claim is true that African rhythm does not have hierarchical meter, then these special rhythms (among all African rhythms) should be those most unlikely to possess hierarchical meter. Good candidates for this purpose are asymmetric timelines (usually played with a variety of bells and high-pitched wooden sticks) because they tend to utilize onsets in positions that are non-metrical (Agawu 2006; Anku 2002a; Anku 2002b; Chernoff 1979; Kubik; Leake 2009; Locke 1982; London 1995; Pressing 1983; Rahn 1987; Rahn 1996; Temperley 2000; Toussaint 2013). To complement the African timelines with a very different African group of rhythms, the pulse saliency histograms of three traditional songs from Ghana notated by Kwabena Nketia (1963a) were calculated and tested. Although Nketia's book contains a score of notated songs, only three have duple meters that could be incorporated into a 16-pulse cycle for fair comparison with the other data (all the timelines used in this study have 16-pulse cycles, whereas the remaining songs in Nketia's book have triple meters that do not divide evenly into sixteen pulses). For the Western rhythms, samples were used for which the pulse saliency histograms were easily available or computable. These pieces span Renaissance and common-practice music, and include Palestrina's *Pater Noster* (Veltman 2006), German folk songs (Huron and Ommen 2006), and compositions by J. S. Bach, Mozart, Brahms, and Shostakovich (Palmer and Krumhansl 1990). For comparison with music theory, the histogram determined by the GTTM hierarchy (see more below) was used. Since the GTTM hierarchy is uniquely defined for a 16-pulse timespan (cycle, measure), the music samples selected for this study all had 16-pulse cycles, thus providing a sharper focus for the comparisons.

The concept of meter has already been used to compare African with Western music (Toiviainen and Eerola 2003). However, as with the concept of rhythm, the music literature offers an abundance of definitions of the term meter (Kvifte 2007; Toussaint 2013). Martin Clayton disentangles some of these definitions (Clayton 1996), Christopher Hasty illuminates the interface between meter and rhythm (Hasty 1997), and Justin London clarifies the distinction between meter and grouping (London 1995 and 2012). On one end of the conceptual spectrum, meter is defined as a mere pulsation of equally spaced (regular) beats (lacking any hierarchy) that may be sounded or merely felt, and that functions as the railing on which rhythms ride. At this level meter divides the timespan cycle (measure) into a specific number of regular beats such as 3, 4, 5, 6, 7, 8, 9, 12, 16, etc., without placing emphases (accents) on any one beat. At the other end of this spectrum, the regular beats are hierarchically arranged according to their strength within an evenly divisible periodic cycle. Such is the view of Lerdahl and Jackendoff (1983), who define meter as a regular pattern of alternating strong and weak beats arranged in a specific hierarchical manner. According to this definition the meter furnishes the musician with a hierarchy of temporal reference points. This structure, referred to as the *Generative Theory of Tonal Music* (GTTM) hierarchy, is illustrated in Figure 1 for the case of a 16-pulse measure (periodic cycle). The height of the column in each pulse position reflects the relative strength of each pulse. The values of these heights should not be interpreted as absolute numerical quantities, but rather as relative magnitudes with respect to each other. What is more important for characterizing the nature of the GTTM hierarchy is the discrete ordering (rank) of these 16 magnitudes. This is the definition of meter that has been frequently invoked to contrast African with Western rhythm, and which is the focus of this study. For the purpose of the statistical analyses carried out here, the magnitudes may be scaled so that they all sum to 1, and thus may be conveniently considered as probabilities of the occurrences of onsets at each of the 16 pulse positions, although such an interpretation is motivated more by Palmer and Krumhansl's empirical finding (1990) than by Lerdahl and Jackendoff's theory (1983).

The GTTM model received considerable criticism regarding its applicability to non-Western music immediately following its publication, mainly from music psychologists and ethnomusicologists (Arom 1991; Belinga 1965; Koetting 1970; Hansen 2011; Magill and Pressing 1997; Nketia 1963b; Tan, Pfordresher, and Harré 2010). One specific criticism has been that the GTTM hierarchy is based on intuition and music theory principles that were not supported by psychological experimental data (Hansen 2011). This criticism spawned several empirical studies to evaluate GTTM's psychological reality (Deliège 1987; Dibben 1994; Palmer and Krumhansl 1990; Todd 1994). In studies with Western music it was found experimentally that the strength of a pulse location correlates well with the degree of the expectancy of occurrence of an onset at that particular location (Palmer and Krumhansl 1990). Even so, a second criticism of GTTM has been that it is a theory applicable only to Western tonal music, and that its claims of universality have not been supported by intercultural research (Hansen 2011). Some writers contend that African rhythms in general, and timelines in particular, exhibit an additive structure rather than being hierarchically evenly divisible, as specified by

the GTTM model (Tan, Pfordresher, and Harré 2010, 292). Indeed, some evidence for this view has been reported by Magill and Pressing (1997, 195) who obtained empirical results suggesting that under certain conditions the asymmetric non-isochronous timeline-ground model (TLG) achieved superiority over the Western isochronous pulse-ground (PG) model. They conclude that the African cognitive model is "subtle, context-dependent, and linked to specific training regimes." For a discussion of the differences between additive and divisive representations of rhythm see Agawu (2003, 86–96), and Entsua-Mensah (2015, 167).

In addition to the above criticisms of the GTTM hierarchy promulgated by music psychologists, some ethnomusicologists have dismissed its applicability to African music. Regarding the role of meter in Sub-Saharan African music, Simha Arom (1991, 206) writes that "the pulsation is the only temporal reference the musicians have." Nketia (1963b, 10) emphasizes that "the African learns to play rhythms in patterns." Such a sentiment is echoed by James Koetting (1970, 124), who writes that "African drummers do not think in terms of meter." M. S. Eno Belinga (1965, 18) dismisses meter outright: "In African music only one thing matters: the periodic repetition of a single rhythmic cell." Of course, learning to play rhythms in patterns, repeating rhythmic cells, and drumming without thinking in terms of meter are not activities that *per se* necessarily produce rhythms that lack meter. Nevertheless, the consensus of these and other authors is that African music does not possess meter in the hierarchical sense embodied by the Lerdahl-Jackendoff model.

The criticisms of GTTM described above, by both music psychologists and ethnomusicologists, are based on the behavioral acts of perception and production of musical rhythms. However, instead of focusing only on the *subjective process* of generating and perceiving African rhythms, we can also seek answers to the aforementioned questions by analyzing the *objective product* instead, *i.e.*, the rhythmic object or written score. In this paper the two questions outlined above, concerning how similar African and Western rhythm are, and whether African rhythm possesses hierarchical meter, are subjected to a mathematical analysis using *pulse saliency histograms*. This yields quantitative and qualitative measures which help to illuminate the structure and degree of this similarity, as well as how much and what type of hierarchical meter African rhythms inform and interact with. Unlike previous comparisons of Western with non-Western music based on objective *acoustic tonal* features (Gomez and Herrera 2008), the analyses presented here are based only on the features of *rhythm* and *meter*, and are restricted to *symbolic notated* music.

As intimated above, the present study uses the methodology of descriptive statistics, rather than a formal, hypothesis-driven approach. In anticipation of criticism from readers who ascribe to such formal methods, an explanatory summary of the issues is warranted. Since the GTTM hierarchy implies a ranking of the 16 pulse frequencies (expectations), with the first pulse receiving the highest rank, the ninth pulse the second highest rank, the fifth and thirteenth tied for the third highest rank, etc. (see Figure 1), a natural measure to compare the association between pulse-frequency histograms is via the Spearman rank correlation coefficient (Spearman 1904), which measures how *monotonic* the relation between two
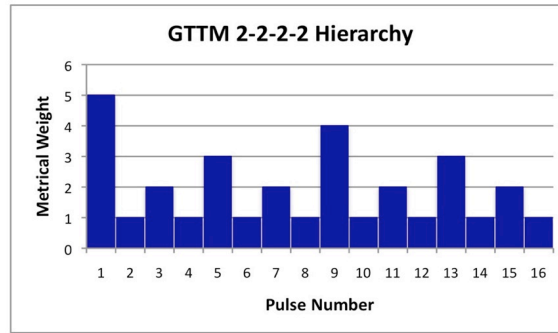
Figure 1. The GTTM metric hierarchy of Lerdahl and Jackendoff.

variables is. Another common measure of association between two variables is the Pearson correlation coefficient (Pearson 1920), which measures how *linear* the relation is. Which of the two correlations is a better description of the association for a particular problem is part of an ongoing debate (Maturi and Elsayigh 2010). Previous studies indicate that sometimes the Spearman correlation is higher than the Pearson correlation, and vice-versa (Hauke and Kossowski 2011). The same behavior was observed with the rhythm data analyzed here, and for this reason, rather than reporting only one of these two correlation coefficients, in the present study both are included. A second issue related to reporting correlation coefficients in descriptive statistics concerns the reporting and meaning of the *p*-values (also called, misleadingly, the levels of significance). This matter also has a long history of debate. Some researchers (Jekel 1977) have asked the question "Should we stop using the *P* value in descriptive studies?" Paraphrasing Perry de Valpine (2014), the purpose of *p*-values is to convince others that either a pattern discernible in data is real, or it could plausibly have arisen by chance alone. More specifically, the *p*-values are probabilities often used in a hypothesis-driven context to test how confident one can be that a statistic calculated from a sample of data taken from a larger dataset (the population) also applies to the population as a whole. It is usually assumed that the smaller this probability is, the more confident we can be in the veracity of the hypothesis. However, this must be taken with a grain of salt since *p*-values are not fixed values, but random variables, and thus have a distribution (Murdock et al. 2008). Concerning the 34 African asymmetric timelines used in this study (see the Appendix), the question then arises as to whether they represent a random sample of some larger population. The purpose of this study is to use the 16-pulse asymmetric timelines used in music from Sub-Saharan Africa and the African diaspora as a worst-case litmus test. These 34 timelines represent *all* the timelines that the author was able to collect from scholarly books and papers on the subject. Therefore the "sample" may be considered to be the entire population, in which case the *p*-values would be considered to be meaningless in the traditional hypothesis-driven context (which this study is not). The reader may care to formulate various hypotheses in terms of larger populations, such as these 34 asymmetric timelines *plus* those not discovered by the author, or those asymmetric timelines consisting of all possible numbers of pulses (timespans, cycles, measures) such as 6, 8, 9, 12, 16, 18, and 24, or

all rhythms used in the music of the African diaspora. However, in these situations the 34 hand-picked timelines would diverge greatly from a *random* sample. Barber and Ogle (2014) consider the question "To *P* Or Not to *P*?" In the present study *p*-values are listed along with correlations in the spirit of descriptive statistics and data mining, but no interpretations in terms of hypothesis testing are implied by their inclusion. The reader is referred to the complex and varied literature on the interpretation of statistical significance tests, and may interpret them appropriately. To complement the numerical correlation coefficients and *p*-values reported, visual plots of the data and graphs of the histograms are provided, which may reflect the true nature of the relations better than abstracted numbers and statistical significance levels.

Regarding the methodology of correlations, an alternative approach to calculating the correlation coefficients between the pulse saliency histograms (frequency distributions) and the GTTM hierarchy (or other models) is to calculate the correlation value for each of the 34 individual timelines with the GTTM hierarchy, and then perform a *t*-test to determine whether the resulting correlation values are significantly greater than zero. A *t*-test is a statistical test to determine whether a sample of data comes from a population with a specific mean (in the present context the mean correlation is hypothesized to be zero). This approach was also tried in order to compare this procedure with the histogram method, and the results are described in the section below on African timelines.

<div align="center">PULSE SALIENCY HISTOGRAMS IN RENAISSANCE AND COMMON-PRACTICE MUSIC</div>

A pulse saliency histogram calculated from a given corpus (dataset) of symbolically notated music resembles the GTTM hierarchy shown in Figure 1. The difference is that the height of a column in the pulse saliency histogram corresponds to the *empirically observed frequency of occurrence* of an onset in that position of the rhythmic cycle. In this section we compare the pulse saliency histograms of Renaissance and common-practice music with the GTTM hierarchy and the 34 timelines of the Appendix. The pulse saliency histogram of the onsets in Palestrina's sixteenth-century motet, *Pater Noster*, compiled by Joshua Veltman (Veltman 2006), is shown in Figure 2. The correspondence between this histogram and the GTTM hierarchy in terms of the ranks of the saliencies of its pulses is visually striking. In both graphs, pulses 1 and 9 contain the highest and second highest columns, respectively. In both graphs, pulses 13 and 5 come next, and have approximately equal height. The same may be said of the third-level pulses 3, 7, 11, and 15, as well as the fourth-level pulses 2, 4, 6, 8, 10, 12, 14, and 16. This visual comparison is compelling enough in this case, but in order to obtain a quantitative measure of the relationship (similarity) that exists between the hierarchies (or ranks) of the two histograms, correlation coefficients may be computed between the vectors determined by each of the sixteen ordered heights. The resulting Spearman (r_s) and Pearson (r_p) correlation coefficients between the two histograms are $r_s = 0.935$ with $p < 0.00001$ and $r_p = 0.955$ with $p < 0.0001$, respectively (refer to Table 2 on page 14 below for these and other results). The very high correlations between these two histograms provide additional strong

quantitative evidence that the GTTM hierarchy is an accurate model of the metric hierarchy contained in Palestrina motets. Note that the Pearson correlation is higher than the Spearman correlation but has lower *p*-value.

Palmer and Krumhansl (1990) calculated the pulse saliency histograms of a sample of common-practice music (which included one contemporary composer—Dmitri Shostakovich). They chose for their analysis a selection of piano compositions by four composers J. S. Bach, French Suites III (Allemande and Sarabande), IV (Gigue), and VI (Gavotte); W. A. Mozart, Piano Sonatas K310, 311, 545, and 576; Johannes Brahms, Piano Intermezzi Opus 118, No. 1, and 119, Nos. 2, 3, and 4; and Dmitri Shostakovich, Piano Prelude III and Fugues III, VI, and XI. Their selection included music composed in a variety of meters, but all the pieces contained fragments with 16-pulse measures, which yielded the pulse saliency histogram shown in Figure 3. The resulting correlations between GTTM and the Palmer-Krumhansl histograms are $r_s = 0.937$ with $p < 0.000001$ and $r_p = 0.959$ with $p < 0.0001$. Again the very high correlations between these two histograms provide quantitative evidence that the GTTM hierarchy is an extremely accurate model of the metric hierarchy contained in the common-practice music of these composers. Note also again that the Pearson correlation is higher than the Spearman correlation but has a lower *p*-value.

The pulse saliency histograms of the Palestrina motet (Figure 2) and the compositions by Bach, Mozart, Brahms, and Shostakovich (Figure 3) suggest the possible discovery of a characterization of the difference between Renaissance music and common-practice music. The Palestrina hierarchy follows the GTTM model almost perfectly in terms of the ranking of pulse positions; only the relative heights differ. This means that all the even-numbered pulses have almost equal height, the pulses numbered 3, 7, 11, and 15 are almost the same, and pulses 5 and 13 have almost equal height. Even the first and ninth pulses are almost equal. This hierarchy possesses many left-right mirror symmetries because all the levels of the hierarchy are flat. In particular, pulses 9, 5, and 13 show a mountain peak structure with the central pulse 9 dominating. Examination of the Palmer-Krumhansl histogram (Figure 3) reveals that although similar symmetrical structures hold for the two lowest levels, the remaining levels
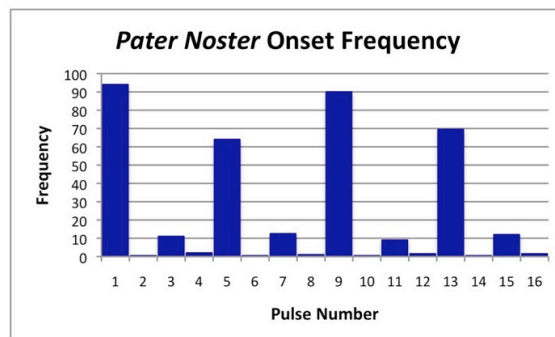

Figure 2. The pulse saliency histogram of Palestrina's Pater Noster (Veltman 2006).
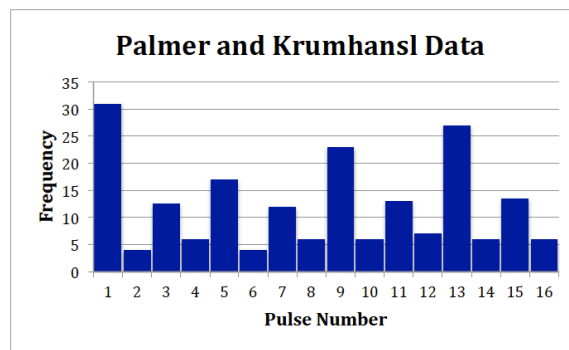
Figure 3. The pulse saliency histogram for 16-pulse timespans of the common-practice music examples compiled by Palmer and Krumhansl (1990).

tell a different story; pulses 5, 9, and 13 show a linear increase with pulse 13 dominating, and thus lack left-right symmetry. This suggests that it may be possible to characterize the difference between Renaissance and common-practice music by the relative strength of pulses 9 and 13. The meter of Renaissance music appears to be more symmetrical than common-practice meter. It would be interesting to confirm this hypothesis with larger samples of these two genres of music. The symmetry properties are analyzed further in the Discussion and Conclusion section below.

A feature that both histograms (Palestrina and the Palmer-Krumhansl data) have in common with each other and with GTTM is that the magnitudes of the ordered pulses are characterized by a perfect up-down alternating pattern. The correlation coefficients between *Palestrina* and the Palmer-Krumhansl histograms are $r_s = 0.933$ with $p < 0.000001$ and $r_p = 0.934$ with $p < 0.0001$.

AFRICAN TIMELINES AND WESTERN MUSIC

The African timelines and the samples of Western music being compared are very different sets of data. Clearly, for the Western music samples there is metrically significant sonic information that is missing from such a comparative analysis. However, this is necessary for a starting point in any reductionist scientific investigation. Future research that takes such additional information into account can then clarify the degree to which the missing metrical information is useful. As already described in the Introduction, apart from a few exceptions, the Sub-Saharan African rhythms that are considered to be the most different from rhythms employed in Western music consist of the *timelines* occurring in drum ensemble music. Much has been written about their uniqueness, complexity, and the amount of syncopation (in relation to Western hierarchical meter) that they possess (Agawu 2006, 2003, and 1995; Arom 1991; Cuthbert 2006; Pressing 1983; Rahn 1996 and 1987; Temperley 2000; Thul and Toussaint 2008; Toussaint 2013 and 2011). Therefore intuition would dictate that these timelines provide a suitable data set (a type of litmus test) to shed light on the claims that African and Western rhythms are fundamentally different, and that African music lacks hierarchical meter. To

inform these claims, a corpus of 34 notated timelines used in the music of Sub-Saharan Africa and the African Diaspora were collected from the following published papers and books (Agawu 2003 and 1995; Arom 1991; Chernoff 1975; Locke 1982; Pressing 1983; Rahn 1987 and 1996; Temperley 2000). These timelines, shown in box notation in the Appendix, all have 16 pulses, and the number k of their onsets (beats) varies between 5 and 10. In this box notation an empty box denotes a silent pulse, and a box containing an X denotes a beat or onset (the strike of the bell, a wooden clave, or a drum). The histogram computed from these 34 rhythms is shown in Figure 4. This histogram looks very different from the one in Figure 1. Apart from the dominant column at pulse number 1, key structural properties of the GTTM hierarchy are not instantly perceivable. In GTTM and *Pater Noster* (Figure 2), the most salient pulses after pulse 1 are 5, 9, and 13, whereas in the African timelines pulses 3 and 7 are preferred, indicating that the latter rhythms are more syncopated than the former. Another obvious difference is that the African timelines make use of most pulses with almost equal frequency, noticeably largely disregarding pulses 2 and 16. However, before hastily concluding that African timelines are profoundly different from Western rhythms, and lack hierarchical meter altogether, it is instructive to compare the hierarchies in these two histograms with the Spearman and Pearson correlations. The correlation coefficients between the African timelines and GTTM are $r_s = 0.793$ with $p < 0.0002$, and $r_p = 0.774$ with $p < 0.0005$. On the other hand, for African timelines and *Pater Noster* the correlations are $r_s = 0.786$ with $p < 0.0002$, and $r_p = 0.588$ with $p < 0.009$. These are high correlations with low p-values. Therefore, although there are some illuminating and important differences between the histograms (discussed further below), what is being tested in addition to the similarity of the overall shape of the histograms, is whether they exhibit similar metric hierarchies as defined in GTTM: alternating strong and weak beats arranged in a hierarchy that exhibits a maximum number of left-right symmetries with the 5-level height constraints of the histogram columns. The Spearman correlation coefficient measures the ordering of the relative heights of the histogram columns, and is thus more sensitive to the contour of the histogram columns than to their absolute values. The contour is a three-symbol string that takes on the symbol "+" if the value increases, "-" if it decreases, and "0" if it stays the same (Marvin 1991; Morris 1993; Quinn 1999). Note that all three histograms have exactly the same contour: [- + - + - + - + - + - + - + - +]. Therefore, the correlation coefficient provides significant evidence that African timelines, as a family, do contain the lower levels of hierarchical meter, and thus are similar in this regard to the rhythms of Western music. The shape of the histogram provides additional useful information as to how they differ.

The histogram for Palestrina's *Pater Noster* has an almost perfect resemblance to the GTTM hierarchy. The histogram of the common-practice music in Figure 13 differs noticeably with the height of column 13, and one may wonder how the histogram might differ if obtained from yet another different European music genre from another century. Thanks to the work of Huron and Ommen (2006), who calculated the onset frequencies for the well-known Essen corpus of Germanic folk songs, this question can be easily answered. The histogram for this corpus, shown in Figure 5, differs somewhat from the histogram of *Pater Noster*, but like the
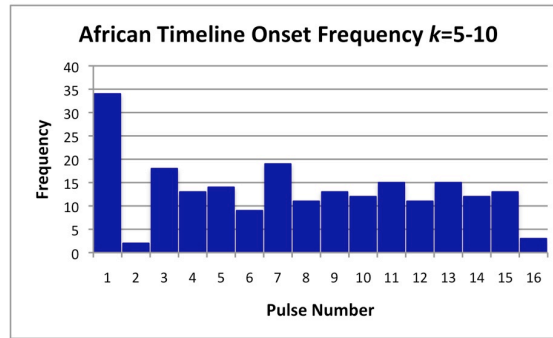
Figure 4. The onset frequencies for the African timelines for k = 5–10.

other four above, has exactly the same contour: [- + - + - + - + - + - + - + - +]. The correlation coefficients between the German folk songs and GTTM are r_s = 0.930 with p < 0.00001 and r_p = 0.973 with p < 0.000001 (similar to *Pater Noster*), and the correlations between the German folk songs and the African timelines histogram are r_s = 0.73 with p < 0.0007 and r_p = 0.680 with p < 0.002. It is worth pointing out a discriminating feature that can be used to distinguish between the German folk songs and Palestrina's *Pater Noster*, which is that in the latter, ignoring the column at pulse 1, the remaining histogram exhibits strong mirror symmetry about pulse 9, so that for example, the heights at the fourth level in the hierarchy, at pulses 3, 7, 11, and 15 are all about the same. On the other hand, for the German folk songs this mirror symmetry is absent: at the third level, pulse 13 is more prominent than pulse 5, and at the fourth level, pulses 7 and 15 are more prominent than pulses 3 and 11. The question of symmetry is discussed in more detail in the Discussion and Conclusion section.

As pointed out in the Introduction, another approach to measuring the association between the African timelines and the GTTM model is to calculate the correlation coefficient for each individual timeline (rather than the frequency distribution of all the timelines) with the GTTM profile, and then perform a *t*-test to determine whether the resulting average correlations are significantly greater than zero (Watkins et al. 2004). The average of the Spearman correlations (averaged over all 34 timelines) is 0.296 with a standard deviation of
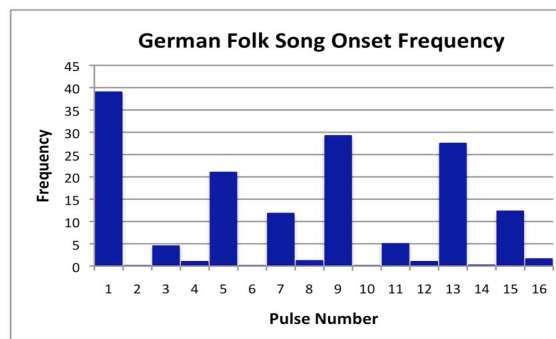

Figure 5. The onset frequencies of the German folk songs (Huron and Ommen 2006).

0.232, a standard error of 0.041, and a *t*-value equal to 7.219 with a *p*-value < 0.0001. In addition, the average Pearson correlation (averaged over all 34 timelines) is 0.320 with a standard deviation of 0.198, a standard error of 0.035, and a *t*-value of 9.151 with a *p*-value < 0.0001. Therefore this alternative method yields results in agreement with the process of comparing the *averaged* timelines in the form of the pulse saliency histograms. Indeed, the Pearson correlations are even higher than the Spearman correlations. From the theoretical point of view, both methods, (1) correlating the averages, or (2) averaging the correlations, are valid methods for measuring the associations between groups, but correlated averages (the method of choice in this study) are more likely to make a point about features of the timelines and how they relate to each other (Monin and Oppenheimer 2005).

African Songs and Western Song

In the comparisons of Western and African musical rhythm described above, the African timelines (bell patterns) are characterized by highly syncopated, short, staccato, sharp sounds made with percussion instruments such as metal bells or wooden sticks. On the other hand, some of the European rhythms were composed of the onsets (beginnings) of sustained notes in songs (*Pater Noster* and German folk songs). One may wonder how the results obtained might change if the onsets of the notes of European songs as well as the GTTM model were compared with the onsets of the notes of African songs (melodies). With such a pilot study in mind, the notated traditional African songs from Ghana (Nketia 1963a) were used to calculate the pulse saliency histograms. Although most of the notated songs listed in Appendix II of Nketia's book (1963a, 115) were composed of a 12-pulse (ternary) meter, three songs incorporated a 16-pulse (binary) meter, and could thus be used for this comparison. These songs included the Akan maiden solo song *Yaw Barima*, the Ga cradle song *Mibi Le Baada*, and the Gonja maiden song *Nayalamu*. The names Akan, Ga, and Gonja refer to the language or cultural group associated with the song. The aggregated onset pulse saliency histogram compiled from the three songs is shown in Figure 6, where the numbers above the seven highest columns indicate their rank according to decreasing height.
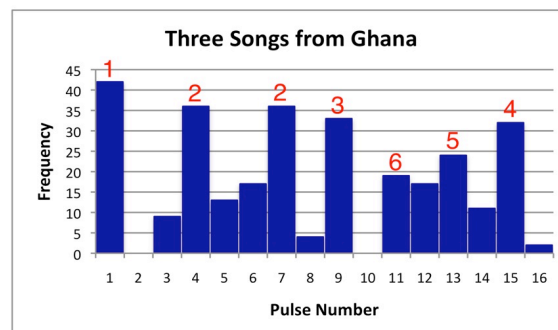

Figure 6. The onset frequencies of the three songs from Ghana.

	GTTM Hierarchy	Palestrina's *Pater Noster*	Common-Practice Music	German Folk Songs	African Timelines
Three Ghanaian Songs	$r_s = 0.596$ $p < 0.00$	$r_s = 0.714$ $p < 0.001$	$r_s = 0.600$ $p < 0.007$	$r_s = 0.633$ $p < 0.005$	$r_s = 0.653$ $p < 0.003$
	$r_p = 0.618$ $p < 0.006$	$r_p = 0.528$ $p < 0.018$	$r_p = 0.610$ $p < 0.006$	$r_p = 0.634$ $p < 0.005$	$r_p = 0.664$ $p < 0.003$

Table 1. Spearman and Pearson correlation coefficients for the three Ghanaian songs.

Table 1 shows the correlation coefficients for the histogram of Figure 6 compared with the GTTM model, as well as the histograms of Palestrina's *Pater Noster*, the samples of common-practice music, the Germanic folk songs, and the African rhythm timelines. All the correlations range from moderately high to high, with values ranging from $r_p = 0.528$ to $r_s = 0.714$ for *Pater Noster*, and all have *p*-values less than 0.02.

The histogram also provides useful information concerning the relative saliency of the locations of the onsets. Consider the seven most frequently used pulse locations, identified in Figure 6 by numbers in decreasing order. Note that pulses 1, 4, and 7 are the three most frequently observed locations, with 4 and 7 tied for second place. By themselves, these three onsets determine the 8-pulse rhythm with inter-onset intervals 3-3-2, one of the most popular 8-pulse rhythms found all over the world (Leake 2009; Pressing 1983; Rahn 1987 and 1996; Toussaint 2013) and known as the *tresillo* in Cuba. This rhythm is also the first half of the distinguished 16-pulse *son* clave rhythm (3-3-4-2-4) (Toussaint 2013). Furthermore, the seven most frequently used pulses determine the rhythm 3-3-2-2-2-2-2, which is the concatenation of the 8-pulse 3-3-2 pattern with the regular 8-pulse pattern 2-2-2-2, and appears in reverse order (2-2-2-2)(3-3-2) as the Central-African timeline in the Appendix. These results suggest that the rhythms of the melodies of these songs are influenced by the rhythms of the asymmetric timelines used in the music. The singing itself emphasizes the rhythms of the timelines that are normally played with a bell. These results provide quantitative validation (of what African musicians and scholars of African music know) that one of the roles of timelines is to organize and inform other layers of rhythmic (and melodic) activity, and that in African and African Diasporic music, timelines act as governing forces that may be felt rather than literally articulated by some player in the ensemble.

To estimate the variability among the three Ghanaian songs, their individual histograms, shown in Figures 7 to 9, were computed. These histograms and the correlation coefficients listed in Table 2 highlight individual differences between the songs, and provide quantitative measures of these differences. The *Nayalamu* Gonja Maiden Song has a histogram that differs significantly from the two other songs, and appears to be uncorrelated with the GTTM model or the German folk songs. These correlations are shown in Table 2. This song correlates mildly with the African timelines ($r_s = 0.452$ with $p < 0.04$, $r_p = 0.582$ with $p < 0.009$), with common-practice music ($r_s = 0.352$ with $p < 0.090$, $r_p = 0.417$ with $p < 0.054$), and with *Pater Noster* ($r_s = 0.450$ with $p < 0.04$, $r_p = 0.380$ with $p < 0.073$). The three histograms have significant

differences from each other. *Yaw Barima* uses only eight of the sixteen pulse locations (as opposed to twelve for the two other songs), and it emphasizes the alternation property, with a violation at pulse 3 so as to start with the 3-3-2 interonset interval pattern. Mibi Le Baada has a regular pattern of adjacent triplets with equally spaced silent pulses at positions 2, 6, 10, and 14. Thus these silent pulses create a shifted 4/4 pattern, creating a prevalent rhythmic pattern from the Colombian *cumbia* music (Toussaint 2013). *Nayalamu* has the distinction of using two groups of five adjacent pulses: 3-7 and 11-15.
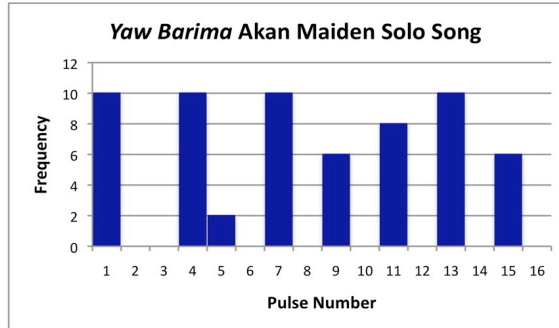
Figure 7. The onset frequencies of the Akan maiden solo song *Yaw Barima*.
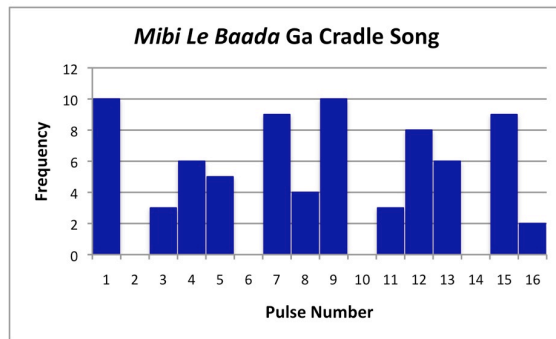
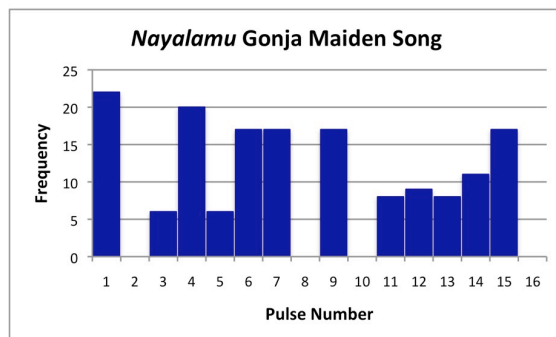**Figure 8.** The onset frequencies of the Ga cradle song *Mibi Le Baada*.

Figure 9. The onset frequencies of the Gonja maiden song *Nayalamu*.

	GTTM Hierarchy	Palestrina *Pater Noster* (Veltman)	Common-Practice Music (Palmer & Krumhansl)	German Folk Songs (Huron & Ommen)	*Yaw Barima* Akan Maiden Song	*Nayalamu* Gonja Maiden Song	*Mibi Le Baada* Cradle Song
African Timelines	$r_s = 0.793$ $p < 0.0002$ $r_p = 0.774$ $p < 0.0005$	$r_s = 0.786$ $p < 0.0002$ $r_p = 0.588$ $p < 0.009$	$r_s = 0.785$ $p < 0.0002$ $r_p = 0.725$ $p < 0.0008$	$r_s = 0.730$ $p < 0.0007$ $r_p = 0.680$ $p < 0.002$	$r_s = 0.740$ $p < 0.001$ $r_p = 0.598$ $p < 0.008$	$r_s = 0.452$ $p < 0.039$ $r_p = 0.582$ $p < 0.009$	$r_s = 0.582$ $p < 0.009$ $r_p = 0.577$ $p < 0.01$
GTTM Hierarchy		$r_s = 0.935$ $p < 0.00001$ $r_p = 0.955$ $p < 0.0001$	$r_s = 0.937$ $p < 0.000001$ $r_p = 0.959$ $p < 0.0001$	$r_s = 0.930$ $p < 0.00001$ $r_p = 0.973$ $p < 0.000001$	$r_s = 0.661$ $p < 0.003$ $r_p = 0.577$ $p < 0.01$	$r_s = 0.374$ $p < 0.08$ $r_p = 0.470$ $p < 0.04$	$r_s = 0.680$ $p < 0.002$ $r_p = 0.659$ $p < 0.003$
Palestrina *Pater Noster* (Veltman)			$r_s = 0.933$ $p < 0.000001$ $r_p = 0.934$ $p < 0.0001$	$r_s = 0.969$ $p < 0.000001$ $r_p = 0.971$ $p < 0.0001$	$r_s = 0.760$ $p < 0.0003$ $r_p = 0.501$ $p < 0.024$	$r_s = 0.452$ $p < 0.040$ $r_p = 0.380$ $p < 0.073$	$r_s = 0.844$ $p < 0.0002$ $r_p = 0.597$ $p < 0.008$
Common-Practice Music (Palmer & Krumhansl)				$r_s = 0.943$ $p < 0.000001$ $r_p = 0.971$ $p < 0.0001$	$r_s = 0.653$ $p < 0.003$ $r_p = 0.645$ $p < 0.004$	$r_s = 0.352$ $p < 0.09$ $r_p = 0.417$ $p < 0.054$	$r_s = 0.753$ $p < 0.0004$ $r_p = 0.658$ $p < 0.003$
German Folk Songs (Huron & Ommen)					$r_s = 0.689$ $p < 0.002$ $r_p = 0.614$ $p < 0.006$	$r_s = 0.361$ $p < 0.09$ $r_p = 0.463$ $p < 0.036$	$r_s = 0.797$ $p < 0.0001$ $r_p = 0.691$ $p < 0.002$
Yaw Barima Akan Maiden Song						$r_s = 0.635$ $p < 0.004$ $r_p = 0.657$ $p < 0.003$	$r_s = 0.702$ $p < 0.002$ $r_p = 0.668$ $p < 0.003$
Nayalamu Gonja Maiden Song							$r_s = 0.612$ $p < 0.006$ $r_p = 0.622$ $p < 0.005$

Table 2. Spearman and Pearson correlation coefficients for the remaining collection of datasets.

DISCUSSION AND CONCLUSION

The results presented here provide some evidence that supports the hypothesis put forward by David Temperley (2000) that African and Western rhythms, at least when compared in terms of the structure of their pulse saliency histograms, exhibit some similarities, although the term "profoundly similar" is perhaps too strong a description if timelines are compared to Western meter. In order to reach such a strong conclusion, additional study should be undertaken using ethnography, a wider all-encompassing set of musical features, and more detailed statistical performance models. Nevertheless, the correlation coefficients provide quantitative measures of the degree of the metric similarity. Furthermore, the structure of the histograms themselves provides insight into the types of

similarities observed. Contrary to several claims found in the literature, the results provide mathematical evidence that African rhythm does possess hierarchical meter, albeit to a limited extent, along the lines of the original GTTM model, thus lending mathematical support to the perspective of Kofi Agawu (1995). The very high correlation between the Renaissance music, common-practice music, and German folk songs with African timelines also adds mathematical support to the view of Mark Hijleh (2008) that the "*practical theory of world rhythm* reveals that even older Western art music can have more in common rhythmically with its Asian, African and American counterparts than might be suspected" (Hijleh 2008).

The descriptive statistical comparisons of the histograms in terms of the Spearman and Pearson correlation coefficients are *global* comparisons. On the other hand, the *local* comparisons of the shapes of the histograms serve to highlight several specific differences between the corpora that are worth noting. From a visual inspection of the histograms in Figures 1 to 5, it is clear that the onsets of the African timelines (Figure 4) are distributed more equally among the 16 pulse locations (with the exception of pulses 1, 2, and 16), whereas in Palestrina's *Pater Noster*, the common-practice compositions, and the German folk songs, the onsets are concentrated on a much more restricted subset of pulse locations. According to the analysis by Veltman (2006, 81), Palestrina's *Pater Noster* is characterized by a 16-pulse cycle, which is adopted here. It would be interesting to perform a similar analysis assuming an 8-pulse cycle to see how much the results differ. A flatter histogram is an indication of the presence of a greater variety of onset placements in relation to the timespan (measure), and thus of increased rhythmic complexity (Thul and Toussaint 2008b). To obtain two quantitative measures of histogram flatness, the Shannon entropy of the histograms (normalized so that they resemble probabilities and sum to 1), and the standard deviations of the histogram bin heights may be calculated by viewing the histograms as discrete probability distributions (Vitz 1968). The entropy has been used before to compare musical styles (Knopoff and Hutchinson 1983; Snyder 1980). Denoting the height (probability) of the *i-th* histogram bin by p_i, the Shannon entropy H was calculated using the formula:

$$H = -\sum_{i=1}^{16} p_i \log_2 p_i$$

The resulting calculations are shown in Table 3, in order of increasing entropy from left to right. The degree of flatness of the histograms is indicated by relatively high values of entropies, and relatively low values of standard deviations. Note that the standard deviations are decreasing monotonically from left to right, and thus both measures are in perfect agreement with respect to the relative complexities of the rhythms of these corpora. From these results we may conclude that Palestrina's *Pater Noster* has the simplest rhythms, and the African timeline rhythms are the most complex (at least by these two measures of complexity). Furthermore, the GTTM profile is more complex than the common-practice rhythms in the Palmer-Krumhansl samples, which are more complex than the rhythms of the

German folk songs compiled by Huron and Ommen, which in turn are more complex than Palestrina's *Pater Noster*. It is worth noting that the maximum possible value of the entropy, attained for a perfectly flat histogram, is 4.0, and that the entropy of the GTTM hierarchy ($H = 3.75$) is almost equal to that of the African timelines ($H = 3.81$). Indeed, both are close to 4.0, again suggesting another commonality between Western and African rhythms. The data also suggest that, on the whole, the meter in Western music has become more complex during the past four centuries, and has reached levels of complexity almost as high as that of the African timelines.

The definition of meter by Lerdahl and Jackendoff (1983), as a regular pattern of alternating strong and weak beats arranged in a specific hierarchical manner, may be untangled in terms of three distinct properties: (1) a regular pattern of alternating strong and weak beats, (2) a particular five-level hierarchical distribution of the relative strengths of these beats, in which the fifth and fourth levels are realized by one pulse location each, the third level is realized by two pulse locations, the second level is realized by four pulse locations, and the first level is realized by eight locations, and (3) the placement of these histogram bins within the timespan (measure) so as to create a fractal pattern with a maximum number of sub-symmetries among the histogram bins of different heights. Informally, a fractal pattern is one that is self-similar at different scales or "looks the same" at different levels of "zooming in" (Mandelbrot 1982). These three properties will be referred to by the shorter terms (1) *alternating property*, (2) *distribution property*, and (3) *fractal symmetry property*, respectively. Note that the third property is a combination of two distinct properties: (a) the presence of sub-symmetries, and (b) whether the sub-symmetries form a fractal pattern. These concepts are illustrated with the GTTM profile in Figure 10. A sub-symmetry of a pattern is a contiguous

	Palestrina's *Pater Noster*	German Folk Songs	Common-Practice Music	GTTM Hierarchy	African Timelines
Shannon Entropy	$H = 2.67$	$H = 2.91$	$H = 3.70$	$H = 3.75$	$H = 3.81$
Standard Deviation	$SD = 0.093$	$SD = 0.081$	$SD = 0.043$	$SD = 0.039$	$SD = 0.033$

Table 3. The Shannon entropies (in bits) of the histograms, and the standard deviations of the histogram column heights, for the GTTM profile and the four rhythm corpora.
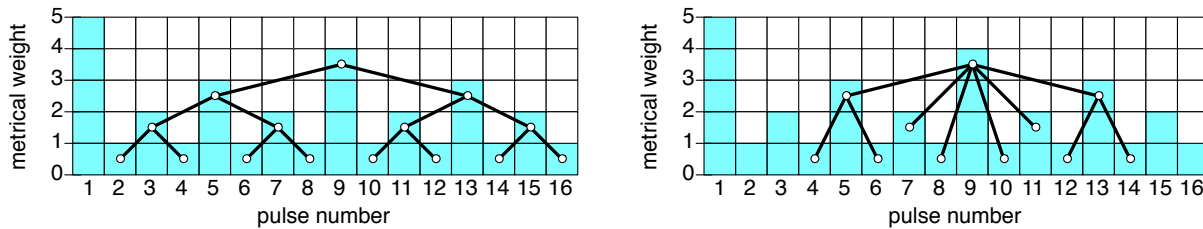
Figure 10. The sub-symmetries present in the GTTM model and their fractal nature. The standard hierarchical fractal sub-symmetries (left), and some additional non-fractal sub-symmetries (right).

sub-pattern that possesses mirror-symmetry, i.e., is palindromic. The number of sub-symmetries contained in a pattern has been shown to be a good predictor of human perception of the complexity of both auditory and visual patterns (Toussaint and Beltran 2013). The entire GTTM pattern from pulses 1 to 16 is not symmetric. However, the sub-pattern from pulses 2 to 16 contains mirror symmetry about pulse 9, and hence is a large sub-symmetry of length 15. However, the GTTM profile contains many more (smaller) sub-symmetries. There are sub-symmetries of length 3 involving levels 1 and 2, such as between pulses 2 to 4, 6 to 8, 10 to 12, and 14 to 16 (shown as leaves of the tree in the left diagram), also involving levels 1 and 3, such as 4 to 6 and 12 to 14 (shown in the right diagram). In the right diagram there is also a sub-symmetry of length 3 between levels 1 and 4, between pulses 8 to 10, and a sub-symmetry of length 5 between pulses 7 to 11, between levels 2 and 4. The reader is invited to discover the remaining sub-symmetries present. What is even more interesting about the GTTM profile is that the sub-symmetries in the left diagram (the standard GTTM hierarchy) have a fractal nature. In the case of the GTTM sub-symmetries the pattern from pulses 3 to 15 (levels 2–4) has the same structure as the two sub-patterns from pulses 2 to 8 and 10 to 16 (levels 1–3). The sub-symmetries highlighted in the right diagram on the other hand do not have the fractal property.

In the comparison of African and Western rhythm, a natural question concerns the relative prominence of the above features of meter in the families of rhythms under study. After all, it is logically possible for a histogram to reflect a pattern of alternating strong and weak beats without exhibiting any hierarchy other than at the two lowest levels sufficient for distinguishing between strong and weak beats (Figure 11, left). Such a profile represents one extreme of the GTTM definition: it contains the minimum amount of hierarchy (lacks the distribution and fractal symmetry properties) while still preserving the alternating property. It is also possible for a histogram to possess a hierarchy consisting of the same histogram bin heights present in GTTM (the distribution property), but arranged in such a manner as to have no alternations whatsoever between strong and weak beats (Figure 11, right). This profile represents another extreme of the GTTM properties inherent in the definition. Note that there are only two possible profiles with this property: the one in Figure 11 (right) and its mirror symmetric version in which the tallest bin lies at pulse 16 rather than pulse 1. The latter version is not explored in this study since it is a complete inversion of the GTTM profile in which the tallest bin is located at pulse 1. In general a corpus of rhythms will contain a pulse saliency histogram that contains a mixture of all these properties. The correlations between these histograms and those of GTTM and the four corpora are shown in Table 4. For Palestrina's *Pater Noster*, the common-practice examples, and the German folk songs, both the Spearman and Pearson correlations are higher for the GTTM hierarchy than for meters with only the alternation property. On the other hand, for the African timelines the reverse is true for the Spearman correlation: for pure alternations $r_s = 0.845$ with $p < 0.00002$, whereas for the GTTM hierarchy $r_s = 0.793$ with $p < 0.0002$. These results suggest that meter in Western music courts the distribution property of GTTM, whereas the timeline ground model of meter in
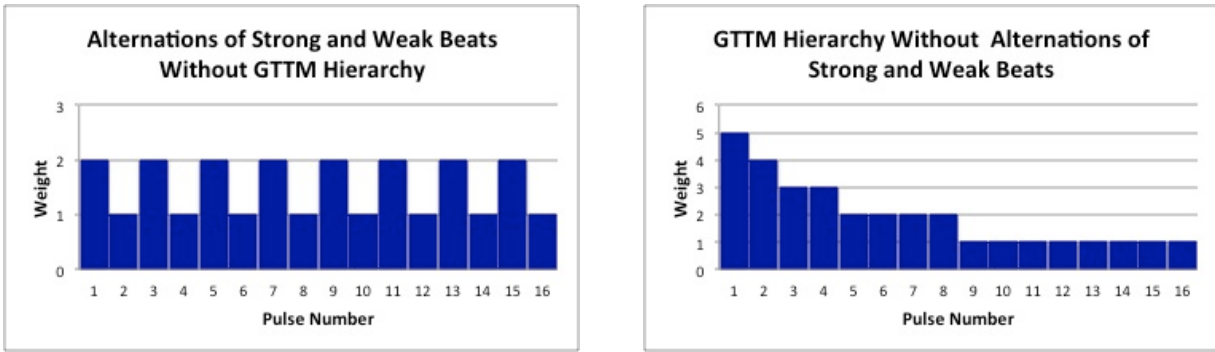
Figure 11. The weights for regular alternations between strong and weak beats (alternation property) without possessing the GTTM distribution property (left), and the GTTM weights arranged to have the distribution property without any alternations (right).

	GTTM Hierarchy	Palestrina's *Pater Noster*	Common-Practice Music	German Folk Songs	African Timelines
GTTM Hierarchy	- - - -	$r_s = 0.930$ $p < 0.00001$ $r_p = 0.955$ $p < 0.0001$	$r_s = 0.937$ $p < 0.000001$ $r_p = 0.959$ $p < 0.0001$	$r_s = 0.930$ $p < 0.00001$ $r_p = 0.973$ $p < 0.000001$	$r_s = 0.793$ $p < 0.0002$ $r_p = 0.774$ $p < 0.0005$
Strong-Weak Alternations without GTTM Hierarchy	$r_s = 0.935$ $p < 0.000001$ $r_p = 0.783$ $p < 0.0002$	$r_s = 0.874$ $p < 0.000005$ $r_p = 0.663$ $p < 0.003$	$r_s = 0.881$ $p < 0.000003$ $r_p = 0.795$ $p < 0.0001$	$r_s = 0.871$ $p < 0.000006$ $r_p = 0.740$ $p < 0.0005$	$r_s = 0.845$ $p < 0.00002$ $r_p = 0.618$ $p < 0.006$
GTTM Distribution Property without the Alternation Property	$r_s = 0.069$ $p < 0.399$ $r_p = 0.302$ $p < 0.127$	$r_s = 0.074$ $p < 0.392$ $r_p = 0.210$ $p < 0.216$	$r_s = -0.123$ $p < 0.326$ $r_p = 0.192$ $p < 0.239$	$r_s = -0.034$ $p < 0.450$ $r_p = 0.236$ $p < 0.190$	$r_s = 0.186$ $p < 0.245$ $r_p = 0.428$ $p < 0.050$

Table 4. Spearman and Pearson correlation coefficients for alternations without the GTTM hierarchy (second row), and for the GTTM hierarchy (with the distribution property) without alternations (third row). For comparison the first row provides the correlations with the original GTTM profile.

African music reflects more the alternation property of GTTM. This is visually evident from the histogram in Figure 4.

Examination of the pulse saliency histogram of the African timelines in Figure 4 suggests the following hierarchy. Apart from pulse 1, which dominates all other pulses by far, six other pulses stand out from the remaining pulses: two pulses are almost nonexistent (pulses 2 and 16), and four are notably higher than the rest: pulses 3, 7, 11, and 13. Pulse 7 is the

second highest, pulse 3 is the third highest, and pulses 11 and 13 are tied for fourth and fifth positions. These observations suggest two modifications of the GTTM model that may be more accurate as theoretical models of the metrical hierarchies of asymmetric African timelines. The first modification is obtained by shifting the columns of GTTM to match the heights of the African timelines as much as possible, while maintaining both the distribution and alternating properties of GTTM. Thus column 9 of GTTM moves to position 7, column 5 of GTTM moves to position 3, and the two lowest levels of the hierarchy are placed so as to maintain the alternating property of GTTM. The resulting metric hierarchy is shown in Figure 12 (left). The correlations with the African timeline profile are shown in Table 5 with the title "GTTM Distribution with Timeline Placements" ($r_s = 0.908$ with $p < 0.000001$ and $r_p = 0.861$ with $p < 0.0001$). The second modification is obtained by relaxing the distribution constraint such that the heights of the columns may be altered by discrete units, but the total number of discrete units used to define the GTTM weights (which number 31) remains the same. The GTTM model may thus be viewed as a distribution of 31 unit squares among the 16
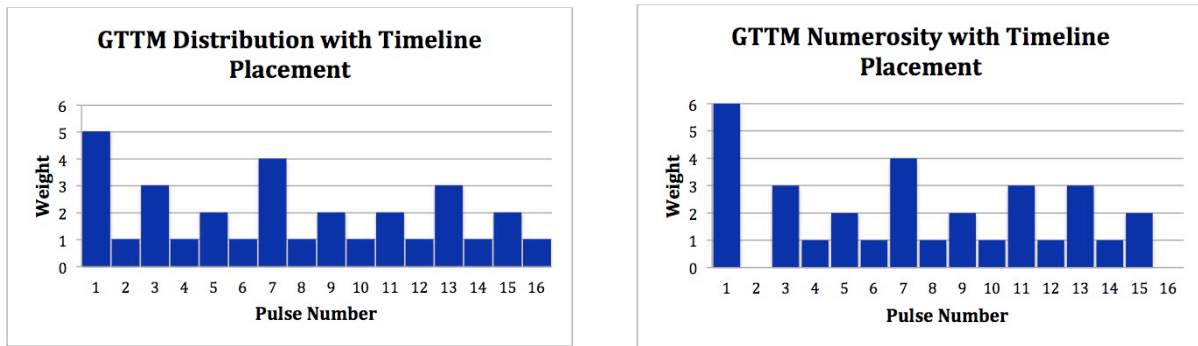

Figure 12. The weights for the GTTM distribution property with GTTM columns swapped to match the African timelines profile (left), and the weights adjusted to have the numerosity property to match the African timelines profile (right).

	GTTM Hierarchy without Alternations	Original GTTM Hierarchy	Strong-Weak Alternations without GTTM Hierarchy	GTTM Distribution with Timeline Placements	GTTM Numerosity with Timeline Placements
African Timelines Histogram	$r_s = 0.186$ $p < 0.245$	$r_s = 0.793$ $p < 0.0002$	$r_s = 0.845$ $p < 0.00002$	$r_s = 0.908$ $p < 0.000001$	$r_s = 0.958$ $p < 0.000001$
	$r_p = 0.428$ $p < 0.050$	$r_p = 0.774$ $p < 0.0005$	$r_p = 0.618$ $p < 0.006$	$r_p = 0.861$ $p < 0.0001$	$r_p = 0.941$ $p < 0.0001$

Table 5. Correlations of the African timelines profile with the GTTM profile and its four modifications.

pulse positions distributed according to several constraints regarding alternations, height columns, and fractal symmetries. This property, the cardinality of the unit squares, is here dubbed the *numerosity* of the profile. The second modification of the GTTM profile makes the minimum number of changes to the distribution constraint with timeline placement while maintaining the numerosity and alternation properties, and is shown in Figure 12 (right) titled "GTTM Numerosity with Timeline Placement." Comparing the African timeline profile (Figure 4) with the profile of Figure 12 (left) shows that the main differences are that pulses 2 and 16 are almost zero, and pulse 1 is considerably higher than all other pulses. The profile in Figure 12 (right) is obtained from the profile on the left by moving the unit square from pulse 2 to pulse 1, to increase its domination, and moving the unit square from pulse 16 to pulse 11 in order to match the heights of pulses 11 and 13.

It is worth noting in passing that as a byproduct of calculating both the Spearman and Pearson correlation coefficients in all the comparisons, numerous examples have been uncovered for which one is greater than the other, and vice-versa, where furthermore, a lower correlation sometimes yields a higher *p*-value, and vice-versa. These observations support the conclusions of Hauke and Kossowski (2011) concerning these two measures of correlation by providing new evidence from data in the domain of musical rhythm. However, in spite of the fact that the Pearson correlations are greater than the Spearman correlations in many of the tests, for the present application the Spearman rank correlation appears to be a more appropriate measure, since monotonic relations are more sensitive to hierarchical rankings than linear relations.

Correlations between the pulse saliency histograms of rhythm corpora and the GTTM (or any other model), as well as entropy values, are of course not the only ways to measure the similarity between metrical hierarchies. Furthermore, these measures do not assess the amount of symmetry or depth present in the metric hierarchies. Visual inspection of the histograms themselves provides additional complementary insights into the structure of the African timeline hierarchy of Figure 4. The modification of GTTM in Figure 12 (right) that preserves the numerosity and alternation properties of GTTM, while maximizing the matching of the placements of the histogram columns at the expense of violating the distribution and fractal properties, is reproduced in enhanced form in Figure 13, with the addition of indications (in thick bold lines) of some of the remaining sub-symmetries and the remaining partial fractal structure. From a comparison of the GTTM histogram with the histogram in Figure 13 it is clear that GTTM has many more sub-symmetries. Table 6 lists the number of sub-symmetries of each length. GTTM has more short sub-symmetries of lengths 3 and 5, and the African timeline profile has no long sub-symmetries of lengths 11, 13, and 15. Furthermore, the 4-level fractal in Figure 10 (left) between pulses 2 and 16 has been reduced to the 3-level fractal in Figure 13 (left) between pulses 4 and 10. It is well known that African timelines possess fewer symmetries than do Western rhythms. That is how they acquired the name "asymmetric" timelines (Rahn 1987). However, the number of sub-symmetries and the size of the fractals present in the pulse saliency histograms are two features that quantify the asymmetries. Symmetry has been used extensively in musical composition for centuries, but it

Length of Sub-Symmetry	3	5	7	9	11	13	15
Number of Sub-Symmetries in GTTM Profile	7	3	2	1	1	1	1
Number of Sub-Symmetries in African Timeline Profile	6	2	2	1	0	0	0

Table 6. Number of sub-symmetries of varying lengths in GTTM and the idealized hierarchical meter for the African timelines.
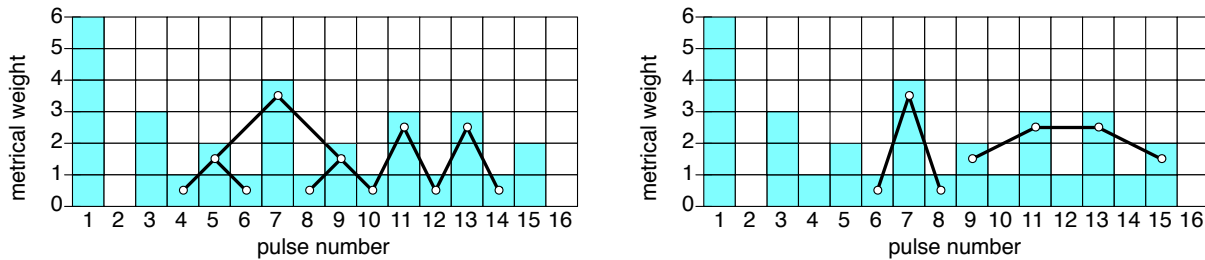
Figure 13. The idealized hierarchical meter model for the African timelines obtained by modifying GTTM while preserving the numerosity and alternating properties of GTTM and maximizing the timeline placements, at the expense of violating the distribution property and losing some sub-symmetries and fractal structure.

is more difficult for the listener to perceive symmetries in the aural domain than for the composer to perceive them in the visual domain (Handel 2006). However, too much symmetry becomes uninteresting, and therefore much attention has been given to striking a balance between perfect symmetry and a more aesthetically pleasing asymmetry, or "broken" or "crippled" symmetry, as it is sometimes called (Anderson 1972; Feldman 1981; Don et al. 2010). For African rhythm it appears that the GTTM model is too symmetrical to correlate well with the timelines profile, and thus the "broken" symmetry in the profile of Figure 13 appears to be a better fit.

Another noteworthy feature of the African timeline profile is the almost complete absence of onsets at pulses 2 and 16. The reason for this may be that since one of the main functions of timelines is their time-keeping role, the musicians need a clear signal as to where the beginning of the cycle (pulse No. 1) falls. This fundamental downbeat signal will be more perceivable (stand out from the crowd of beats) if it is surrounded on both sides by a silent pulse (pulse 2 just after the main beat, and pulse 16 just before). However, the same behavior is observed in the songs from Ghana, the Renaissance music example, and the German folk songs. Indeed only the common-practice music examples make use of these two pulse positions. It may be that the same reasoning applies to the German folk songs, but this is speculation that invites further research. It does not apply to Palestrina's *Pater Noster*

represented in a 16-pulse cycle because this piece tends to ignore almost completely all the even-numbered pulses.

Examination of the various properties of the GTTM definition helps to further characterize the differences between the Renaissance music of Palestrina, the common-practice music samples compiled by Palmer and Krumhansl, and the German folk songs in the Essen collection studied by Huron and Ommen. The pulse saliency histogram of Palestrina's *Pater Noster* (Figure 2) is almost identical to the GTTM model in the sense of possessing all the fractal sub-symmetries of GTTM. The main difference is in terms of the relative difference in the heights of the histogram bins. For *Pater Noster* the even pulses are almost not used at all and pulses 3, 7, 11, and 15 are much less used than pulses 5, 9, and 13. It would be interesting to determine if this property holds for Palestrina's music in general, or even for Renaissance music as a whole.

The empirical pulse saliency histogram of the common-practice samples (Figure 3) may be idealized to the hierarchical meter model shown in Figure 14 (left). Apart from the height of the histogram bin at pulse 13, the remaining profile is for all practical purposes identical to that of GTTM. However, the fact that the height of pulse 13 is higher than pulse 9 alters the number of sub-symmetries present considerably, and destroys the fractal nature of the hierarchy, while preserving the alternation property. Interestingly the three heights of pulses 5, 9, and 13 are collinear, introducing a different type of regular structure that compensates for losing some of the symmetries. This pattern suggests that common-practice music places more emphasis on the last (fourth) quarter note than on the third quarter note in the middle of the timespan, as do Palestrina's *Pater Noster*, the German folk songs, and GTTM, where the quarter note at pulse 9 is used more frequently than the quarter notes at pulses 5 and 13. To speculate further, a tantalizing similarity is suggested between the frequency patterns of these three quarter notes and the accents (stresses) of three-syllable words in the English and French languages. English tends to stress the middle syllable (as in the English word "production"), whereas French tends to place the stress on the third syllables (as in the French word "production"). The metric hierarchy of the common-practice music appears to match
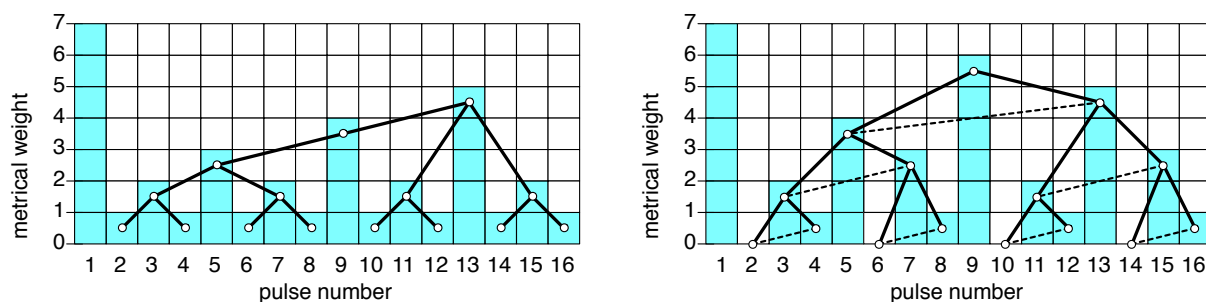
Figure 14. The idealized hierarchical meter models for the common-practice music examples of Palmer and Krumhansl (left), and the German folk song data of Huron and Ommen (right).

the French pattern of syllable stresses, whereas the metric hierarchies of *Pater Noster*, the German folk songs, and GTTM appear to match the English syllabic stress pattern. The commonalities of rhythm in language and music have been investigated previously (Huron and Ollen 2003; Patel and Daniele 2003a and 2003b). However, these authors compared the complexity of English and French language with English and French music using the normalized pairwise variability index (nPVI) as a measure of rhythm complexity. It would be interesting to determine if there also exist relations between music and language with respect to the location of the stresses in musical meters and words, respectively. More specifically, the fact that the German folk songs place emphasis towards the end of the cycle, akin to the French accents in language, raises the perhaps strange question of whether the French language influenced the meter of common-practice music?

The empirical pulse saliency histogram of the Essen German Folksong Collection (Figure 5) may be idealized to the hierarchical meter model shown in Figure 14 (right). This hierarchy is uniquely different from GTTM and the profiles of all the other pulse saliency histograms examined here, in that it contains none of the left-right mirror sub-symmetries which are a hallmark property of GTTM, while still completely preserving the alternation and fractal properties of GTTM. The sub-symmetries are broken by the fact that, at every level of the hierarchy that connects a "parent" pulse to its two "offspring" pulses, the height of the offspring on the right is higher than the offspring on the left (indicated in the figure by dashed lines). Thus, for pulses 2, 3, and 4, the height of column 4 is greater than the height of column 2. Similarly, for pulses 6, 7, and 8, the height of column 8 is greater than the height of column 6. For pulses 3, 5, and 7, the height of column 7 is greater than the height of column 3. At the highest level (for pulses 5, 9, and 13) the height of column 13 is greater than the height of column 5, and so on. It is as if the GTTM hierarchy has been rotated in a counterclockwise direction by some angle. Therefore in spite of this "rotation" the hierarchy completely preserves the fractal property of GTTM. Furthermore, although the hierarchy does not possess the left-right mirror sub-symmetries, it does possess more complex sub-symmetries that involve simultaneous translations in a northeasterly direction accompanied by a dilation. Thus the triangle determined by pulses 6, 7, and 8, is larger than the triangle determined by pulses 2, 3, and 4. Similarly, the triangle determined by pulses 11, 13, and 15, is larger than the triangle determined by pulses 3, 5, and 7. These types of symmetries imply that the German folk songs are rhythmically more complex than the rhythms of Renaissance and common-practice music, as well as those implied by the GTTM profile. They also suggest that the German folk songs tend to place more emphasis on the notes appearing at the ends of the sub-symmetries. It would be interesting here also to explore the relationship between this structure in music syllable stress patterns in the German language.

Another property of rhythms that may be used as a feature for distinguishing between families of rhythms is the preference for utilizing pulse locations in the first half of the timespan (cycle). This property can be measured by considering the normalized histograms as probability distributions, and calculating their expected values, which are equivalent to the

centers of gravity of the normalized histograms. In a perfectly balanced meter, in which the number of onsets that occur in each half-measure is the same, the expected value should be 8.5. The expected values, in increasing order, are: (1) *Pater Noster* = 7.03, (2) German folk songs = 7.43, (3) African timelines = 7.61, (4) GTTM hierarchy = 7.71, and (5) common-practice music = 8.09. Common-practice music is the closest to being perfectly balanced, the *Pater Noster* has the most notes in the first half-measure, and the African timelines are almost indistinguishable from the GTTM hierarchy. Hence this is another feature of the pulse saliency histograms that the African timelines and GTTM have in common.

The shapes of the histograms also highlight the differences between the rhythms of the Ghanaian and Western songs. These differences are most evident in the first half (eight pulses) of the timespans. The most salient pulses in the *Pater Noster*, the common-practice music samples, and the German folk songs are the first, fifth, ninth, and thirteenth pulses, whereas in the Ghanaian songs they are the first, fourth, seventh, and ninth. The inter-onset intervals of the former are 4-4-4, whereas as for the latter they are 3-3-2. These differences are an indication that the rhythms of the Ghanaian songs tend to be more syncopated than their Western counterparts, and serve to distinguish African music from Western music more ably than the possession of hierarchical meter. While the results of the pilot study with the three songs from Ghana are suggestive, they clearly represent a considerably limited and unrepresentative sample of the traditional music of Ghana, let alone the music of Sub-Saharan Africa. Not only is a sample of size three rather small, but also, these songs belong to the genres of maiden songs and cradle songs, and as the histograms show, they exhibit considerable differences from each other. Similar tests should be made with much larger corpora of songs belonging to a greater variety of genres before more general conclusions can be made about the rhythmic aspects of the melodies of traditional African songs.

ACKNOWLEDGMENTS

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APPENDIX

The 34 African, Afro-Cuban, and Afro-American rhythm timelines used in this study are shown in box notation. Each pulse (box) takes on one unit of time. The marked boxes are sounded, and the empty boxes are silent pulses (rests). The names used to identify these timelines are for convenience only, and although they are commonly found in Western media, no cultural ownership or chronological priority is intended by their application.

Pulse-Number	1	2	3	4	5	6	7	8	9	1	1X	12	13	14	15	16
Bossa-1	X			X			X				X			X		
Bossa-2	X			X			X		X				X			
Bossa-3	X			X			X		X					X		
Shiko	X				X		X				X		X			
Timini	X		X				X		X				X			
Kromanti	X		X				X				X		X			
Tuareg	X				X		X		X				X			
Tipitina	X		X		X					X					X	
Son	X			X			X				X		X			
Rumba	X			X				X			X		X			
Gahu	X			X			X				X				X	
Domba	X		X			X				X				X		
Kpatsa	X				X				X			X		X		
Rap-X	X				X			X		X			X			
Soukous	X			X			X				X	X				
Rap-2	X		X					X			X				X	
Mambo-1	X			X		X			X		X			X		
Mambo-2	X		X			X			X			X		X		
R-W-B	X		X			X			X			X			X	
Popcorn	X		X		X			X		X					X	
Funky	X		X		X			X		X			X			
Central-Africa	X		X		X		X		X				X		X	
Takoe	X			X		X			X		X		X		X	
Akom	X		X		X		X			X		X			X	
Adangme	X		X		X		X		X			X		X		
Samba	X		X			X		X		X			X		X	
Ghana	X		X		X		X			X		X		X		
Bembe-duple	X			X			X	X			X		X			X
Oyaa	X		X		X		X		X		X	X		X	X	
Ngbaka	X		X		X		X	X		X		X		X	X	
Ngbaka-Maibo	X		X		X		X	X		X		X		X		X
Kassa	X	X		X		X	X		X		X		X		X	
Mutuashi	X	X		X		X		X	X		X		X		X	
Rumba-Palitos	X		X	X		X		X	X		X		X	X		X