Response to Mirelman: Orality and Aristoxenus; Pedagogy and Practice

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When Dr. Mirelman submitted his account of Mesopotamian tuning (roughly 1850–600 BCE, in a region extending from present-day Iraq to Syria) for this journal (vol. 2, no. 2, 2013), he suggested that I elaborate on topics that arose in the discussion period after his presentation at the Second International Conference on Analytical Approaches to World Music in 2012. These included general relationships between Mesopotamian tuning and orality, which I construe here broadly as including aurality—that is, Mesopotamian tuning's basis in auditory perception. Also considered below are comparisons of tuning in Mesopotamia with Aristoxenus's similarly perceptual approach to tuning in ancient Greece. Introduced here as well are novel conjectures concerning the place of an important source (CBS 10996) for Mesopotamian tuning in pedagogy and practice, and what I believe is the corroborating evidence provided by the compositional design of the only surviving notations of Mesopotamian music.

ORALITY

Mirelman's (2013, 54–55) emphasis on the importance of orality has considerable bearing on his discussion of Mesopotamian tunings or what studies of Mesopotamian music have termed "modes." Scholarly writings on Mesopotamian music have often used the terms "tuning" and "mode" interchangeably. In other musicological research, the term "mode" has been employed very broadly, especially since the publication of the unusually extensive entry on mode by Harold Powers (1980) in the *New Grove Dictionary*. Moreover, studies in comparative musicology, ethnomusicology, music theory, and music cognition have frequently conflated two aspects of tuning, namely, a tuning's acoustical and perceptual characteristics. As what is known of Mesopotamian music does not correspond closely with the use of the term "mode" in musicological writing generally, I employ the term "tuning" in much of what follows. Further, as countering the widespread conflation of acoustical and perceptual aspects of tuning is important for understanding Mesopotamian tunings within a framework of orality/aurality, I begin by distinguishing between them.

The acoustical aspects of a tuning are a matter of measurement—for example, the fundamental frequencies of tones are measured in cycles per second (i.e., Hertz, abbreviated Hz) and their spectral features are measured in Hertz (Hz), milliseconds (ms), and decibels (dB). Of general relevance to a tuning are the intervals between pairs of its tones, which are calculated in terms of ratios or proportions––for example, 3/2 or 3:2 for a pair of tones comprising A4 = 440 Hz and E5 = 660 Hz (i.e., an ideal Pythagorean [or just, limit-3] perfect 5th above A4).

Since seminal publications by Alexander J. Ellis and Alfred J. Hipkins more than a century ago (Ellis and Hipkins 1884, 369–71, 372ff.; Ellis 1885, 498ff.), such ratios or proportions have been transformed logarithmically into twelve-hundredths of an octave (e.g., for 3/2 or 3:2, the logarithm of 3/2 to the base 2^(1/1200), or in an Excel spreadsheet, log(3/2,2^(1/1200)) = 702 cents, to the nearest twelve-hundredth root of 2).

Although there are no Mesopotamian tones that one can now measure in this way, the acoustical concept of an interval as a ratio or proportion of two fundamental frequencies remains valuable because of the well-established relationship between such ratios or proportions and the way in which people respond to them perceptually (i.e., by ear). In general, if one pair of tones is heard as constituting a larger interval than another pair of tones, the fundamental frequencies of the first pair of tones comprise a ratio or proportion that is acoustically larger than the ratio of the fundamental frequencies of the second pair of tones. As there is no reason to believe that human perception has changed in this regard since Mesopotamian times, one can assume that this relationship was true four millennia ago and one can allow its consequences to inform one's understanding of how Mesopotamian tunings were actually heard by Mesopotamians themselves.

This assumption is of great importance in understanding Mesopotamian tuning not only because there are, as mentioned above, no Mesopotamian tones that can be measured acoustically in cycles per second, but also because the way in which Mesopotamian theory was formulated was implicitly perceptual. As Mirelman (2013, 49) emphasizes, a pair of tones that spanned four or five consecutive strings on a nine-string harp was characterized as *zakû* (i.e., clear, pure, clean, free: Oppenheim 1961, 23–32) or *la zakû* (unclear, impure, unclean, not free: Oppenheim 1973, 1–5), just as one might say a particular traffic light was red or not red. For Mesopotamian intervals, not only is there no question of acoustical measurement, there is as well no question of an abstract, mathematical, numerical formulation. Nonetheless, as discussed below, the very important and relatively recent mathematical formulation of well-formed scales by Norman Carey and David Clampitt (1989, 200–202), if combined with the notion that one interval might be heard as larger than, the same size as, or smaller than another—just as one light might be seen as brighter than, as bright as, or less bright than another light—can clarify considerably what Mesopotamian music sounded like.

Mirelman's (2013, 49–54) translation rightly observes that a particular harp or lyre was considered to be tuned in one of seven particular ways and each of these seven ways was accorded a distinctive name. Avoiding numbers and assuming a widely accepted conclusion about Mesopotamian tuning names (cf. Vitale 1982, 252; Gurney 1994, 103; Gurney et al. 1998, 223–25; Krispijn 2008, 12), one can characterize a nine-string *nīd(i) qablim* harp or lyre (i.e., a nine-string harp or lyre that was heard as being in *nīd(i) qablim* tuning) in terms of smaller (S) and larger (L) intervals between consecutive strings as shown in Figure 1.

letter name:	e		d		c		b		a		g		f		(e_1)		(d_1)
string number:	1		2		3		4		5		6		7		(8)		(9)
interval size:		L		L		S		L		L		L		S		(L)	

Figure 1. Relative interval sizes and example letter names on a nine-string *nīd(i) qablim* harp or lyre.

According to the widely held conclusion just alluded to, the tones produced by the nine strings were successively lower in pitch—that is, the tone produced by string 1 was higher than the tone produced by string 2, the tone produced by string 2 was higher than the tone produced by string 3, and so forth. Because of its resemblance to what was called "diatonic" tuning in later millennia and because this way of tuning a *nīd(i) qablim* harp or lyre would have resulted in tones whose pitches were arranged downward from string 1 rather than upward, this tuning is termed "descending diatonic" below. As well, since the L-L-S-L-L-L-S pattern within an octave would be heard as diatonic, one can regard it as an instance of Easley Blackwood's (1985, 195–200) "recognizable diatonic tunings" (discussed below).

According to yet another aspect of a widely held conclusion about Mesopotamian tuning, string 8 was heard as an octave lower than string 1 and string 9 was heard as an octave lower than string 2. Using the letter names of European music, strings 1 to 9 could thus be labeled as in Figure 1, where the subscripts indicate that strings 8 and 9 (which are labeled e_1 and d_1) are heard as an octave lower than strings 1 and 2 (labeled e and d).[1]

Unlike other writers on Mesopotamian music, Mirelman (2013, 47, 55) astutely notes that strings 8 and 9 might have formed unisons rather than octaves with strings 1 and 2. In Mesopotamian depictions, the strings of harps are arranged evenly from longer to shorter. In order for strings 8 and 9 on such a harp to be heard as forming unisons with strings 1 and 2 they would have to be substantially less massive (i.e., thinner) and/or wound much more tensely than their neighbors. Since no organological or pictorial evidence supports such an interpretation, one can conclude that strings 8 and 9 of Mesopotamian harps formed instead octaves with strings 1 and 2. Nonetheless, in certain depictions of Mesopotamian lyres, strings differ greatly in length so that strings 8 and 9 on such lyres might have been heard as constituting either octaves or unisons with strings 1 and 2. Moreover, Mirelman (2010, 49–51) has recently adduced considerable evidence that favors the conclusion that the nine-string instrument implied by the central tuning sources was a lyre rather than a harp. Whether strings 1 and 8 and strings 2 and 9 were heard as unisons or octaves, they can be understood as having constituted classes of scale degrees. Accordingly, the subscript letter names e_1 and d_1 employed above refer to both possibilities.

[1] Throughout this paper, strings are designated with numerals (e.g., "string 4"), musical intervals are designated with ordinal names (e.g., "perfect 4th"), pitches are designated with lower-case letters, and pitch classes are designated with upper-case letters.

	letter name:	e		d		c		b		a		g		f♯		(e$_1$)		(d$_1$)
a)	**string number:**	1		2		3		4		5		6		7		(8)		(9)
	interval size:		L		L		S		L		L		S		L		(L)	
	letter name:	e		d		c		b♭		a		g		f		(e$_1$)		(d$_1$)
b)	**string number:**	1		2		3		4		5		6		7		(8)		(9)
	interval size:		L		L		L		S		L		L		S		(L)	

Figure 2. Retunings of the instrument in Figure 1 from *nīd(i) qablim* to a) *pītum* and b) *nīš tuḫrim*.

As Mirelman (2013, 49–50) further relates, changing a harp or lyre from *nīd(i) qablim* tuning to another tuning involves tightening or loosening one or more particular strings (or one or more particular pairs of strings if strings 8 or 9 are involved). For example, as Figure 2a illustrates, if string 7 is tightened so that the interval between strings 6 and 7 is S and the interval between strings 7 and 8 is L, the instrument's tuning is changed from *nīd(i) qablim* to *pītum*. Similarly, if string 4 is loosened so that the interval between strings 3 and 4 is L and the interval between strings 4 and 5 is S, the harp or lyre's tuning is changed from *nīd(i) qablim* to *nīš tuḫrim*, as in Figure 2b.

As indicated above, there is no mention of ratios, proportions or measurements in any Mesopotamian account of tuning. Instead, adjusting a string changed its fundamental frequency, so that the string was heard as higher or lower than previously depending on whether the string was tightened or loosened. As well, adjusting a string resulted in changes to the intervals it formed with other strings and changed a harp or lyre from one tuning to another. Figure 3, which assumes descending-diatonic tuning, summarizes the ways in which by tightening or loosening a single string (or on a harp or lyre having more than seven strings, a single pair of strings that formed an octave or a unison) changed the pattern of clear and unclear intervals on a harp and changed the name for the tuning of the harp or lyre.

As Figure 3 shows, the Mesopotamian sources imply that changes to the tuning of a harp or lyre were cyclic, proceeding as they did in increments that corresponded to an interval that spanned four strings and whose size comprised one smaller step and two larger steps, or its inversion (or complement) relative to an octave spanning eight strings, an interval that spanned five strings and whose size comprised one smaller step and three larger steps.

That the European letter names provided for illustrative purposes in Figure 3 do not come full circle is of no consequence for the tuning system. The distinctive aspect of each of the seven named tunings was the pattern of intervals among its seven steps. Notwithstanding the European letter names in Figure 3, the interval pattern among the seven steps of a harp or lyre in *išartum* tuning is, from top to bottom in descending-diatonic: S-L-L-L-S-L-L (Figure 4). As Mirelman (2013, 55) stresses, it is this pattern of relative pitch that is characteristic of *išartum*, not the absolute values of its fundamental frequencies.

tuning name:	**identifying string-pair:**	**unclear string-pair:**	**string numbers and European letter names:**								
			1	**2**	**3**	**4**	**5**	**6**	**7**	**(8)**	**(9)**
išartum	2 & 6	5 & 2	e	*d♯*	c♯	b	*a*	g♯	f♯	(e_1)	*($d♯_1$)*
kitmum	6 & 3	2 & 6	e	*d*	c♯	b	a	*g♯*	f♯	(e_1)	*(d_1)*
embūbum	3 & 7	6 & 3	e	d	*c♯*	b	a	*g*	f♯	(e_1)	(d_1)
pītum	7 & 4	3 & 7	e	d	*c*	b	a	g	*f♯*	(e_1)	(d_1)
nīd(i) qablim	4 & 1	7 & 4	e	d	c	*b*	a	g	*f*	(e_1)	(d_1)
nīš tuḫrim	1 & 5	4 & 1	*e*	d	c	*b♭*	a	g	f	*(e_1)*	(d_1)
qablītum	5 & 2	1 & 5	*e♭*	d	c	b♭	*a*	g	f	*($e♭_1$)*	(d_1)
išartum	2 & 6	5 & 2	e♭	*d*	c	b♭	*a♭*	g	f	($e♭_1$)	*(d_1)*

Figure 3. Patterns of clear and unclear intervals on a nine-string instrument created by tightening or loosening a single string.

string number:	1		2		3		4		5		6		7	(8)
European letter name:														
first row of Figure 3:	e		*d♯*		c♯		b		*a*		g♯		f♯	(e_1)
last row of Figure 3:	e♭		*d*		c		b♭		*a♭*		g		f	($e♭_1$)
interval size:		S		L		L		L		S		L	L	

Figure 4. Tuning pattern of *išartum* in Figure 3.

Taken at face value, the original sources of Mesopotamian tuning assume that a harp or lyre has already been adjusted to conform to the interval pattern of a particular tuning and focus instead on how to change such an instrument from one of the seven tunings to another. However, as Raoul Vitale (1982, 248–49) observed, the tuning sources also imply how one could adjust the strings of a previously untuned harp or lyre "from scratch" so that its intervals would conform to a particular tuning. In this regard, Mirelman's (2013, 44n3, 45–51) observation that the name of a particular pair of strings was the name for a particular tuning (cf. Krispijn 2008, 12) is of considerable significance, for the interval of this pair of strings could well have been employed as the first interval in a cycle that would generate the entire tuning. For instance, *nīd(i) qablim* refers to the interval formed by strings 4 and 1 (a 4th) and to the tuning that is represented in Figure 3 by the string numbers and European letter names in Figure 5a.

One could tune any previously untuned harp or lyre to conform to the interval pattern of *nīd(i) qablim* by adjusting strings 4 and 1 so that the interval they formed was heard as "clear." Thereupon, one could tune the rest of the strings by ensuring that successive intervals up a 4th or down a 5th also were heard as clear. Starting with the interval between strings 4 (b) and 1 (e), the tuning cycle for *nīd(i) qablim* would be as laid out in Figure 5b.

a) The pair of string numbers and letter names that denotes both the interval of a 4th and the tuning *nīd(i) qablim*.

string number:	**1**	2	3	**4**	5	6	7	(8)
letter name:	***e***	d	c	***b***	a	g	f	(e_1)

b) Tuning cycle for *nīd(i) qablim*.

pair of strings:	4-1	1-5	5-2	2-6	6-3	3-7
letter names:	b-e	e-a	a-d	d-g	g-c	c-f

Figure 5. Association between pairs of strings and particular tunings.

For *nīd(i) qablim* tuning, the final string of the cycle would be string 7 (f), the first string of the cycle would be 4 (b), and the interval formed by these strings (f-b) would be heard as "unclear." By comparing the sound of strings 4 and 7 with sound of strings 3 and 7 (or strings 6 and 3 or strings 4 and 1), one could confirm that the other steps had been perceptually accurate. Thereupon, if one tightened string 7, the harp or lyre's tuning would be changed from *nīd(i) qablim* to *pītum* (e-d-c-b-a-g-f♯); if, instead, one loosened string 4, the instrument's tuning would be changed from *nīd(i) qablim* to *nīš tuḫrim* (e-d-c-b♭-a-g-f); and so forth.

As indicated above, the key to the tuning's structure, namely, steps whose S-L-L-S-L-L-L intervals cycle within an octave and from one octave to another, is the fact that if six of seven intervals that span the same number of steps within an octave (or most generally, i-1 of i intervals that span d_g degrees within a modular interval of size s_m that spans d_m [= i] degrees) are the same in size, s_d, the resulting cycle is non-degenerate well-formed (Carey and Clampitt 1989, 200–202; cf. Rahn 2011b, 213–18).

As Mirelman (2013, 49, 54) points out, scholars of Mesopotamian music have assumed that this generating interval was a perfect 4th and its complement, a perfect 5th. Blackwood (1985, 195–200) has shown that, formulated in an abstract, mathematical manner and relative to a similarly abstract mathematical octave whose ratio is s_m=2/1, the size of the generating interval, s_g, for the S-L-L-S-L-L-L cycle could be anywhere between $2s_m/5$ and $3s_m/7$, exclusive (or between $3s_m/5$ and $4s_m/7$, again exclusive).

Such an abstract, mathematical value could be realized only approximately, even if one employed such a device as Ptolemy's monochord, which is not known to have existed until the second century CE. In contrast, a perceptual formulation would emphasize that each of the intervals of size S was heard as smaller than each of the intervals of size L (i.e., S < L). Consistent with such a formulation would be perceptual counterparts to the following inequalities among sums of registrally successive intervals:

S+L < L+L (cf. S+L < 2L: e.g., d_1-f, e_1-g, a-c, b-d *vs* f_1-a, g-b, c-e)

L+L < S+L+L(cf. 2L < S+2L)

S+L+L < S+L+L+S (cf. S+2L < 2S+2L) and

S+L+L < L+L+L (cf. S+2L < 3L)

S+L+L+S < S+L+L+L (cf. 2S+2L < S+3L) and

L+L+L < S+L+L+L (cf. 3L < S+3L)

S+L+L+L < S+L+L+L+S (cf. S+3L < 2S+3L)

S+L+L+L+S < L+L+S+L+L (cf. 2S+3L < S+4L)

L+L+S+L+L < S+L+L+S+L+L (cf. S+4L < 2S+4L)

S+L+L+S+L+L < L+L+S+L+L+L (cf. 2S+4L < S+5L)

Noteworthy in such a formulation is that intervals of a particular size are not necessarily heard as equal in size to each other; that is, S=S, L=L, S+L=S+L, and so on, would not necessarily hold perceptually. In abstract, mathematical terms, which many centuries later would be more or less closely approximated by such a device as the monochord, the ranges of possible values of each kind of interval on its own would be quite considerable.

Figure 6 displays the values that would result if *nīd(i) qablim* were tuned employing the extreme abstract, mathematical values for a clear 4th or 5th. Assuming an octave of size s_m = 1200 cents, these generating intervals' values would be $(2/5)s_m$ = 480 and $(3/7)s_m$ = 514 for the smallest and largest possible clear 4ths, and $(3/5)s_m$ = 720 or $(4/7)s_m$ = 686 for the largest and smallest possible clear 5ths. In Figure 6a, both kinds of extreme generation, proceed from letter name b up a smallest or largest clear 4th to e, down a largest or smallest 5th to a, and so forth, as outlined above. Figure 6b summarizes the ranges for each kind of interval: small 2nd (S), large 2nd (L), small 3rd (S+L), etc.

All of the intervals' cents values correspond to their values in terms of S and L—for example, the maximum value for any small 2nd (S) is smaller than the minimum value for any large 2nd (L), and similarly for the latter in comparison with the minimum value for any small 3rd. Moreover, with a single exception, none of the intervals' ranges overlap. The single exception comprises the large 4ths, which can range from 514 to 720 cents and the small 5ths, which can range from 480 to 686 cents.

These two kinds of interval share the range from 514 to 686 cents as possible values and their overlap is consistent with the Mesopotamian formulation of clear and unclear intervals. On one hand, there is no overlap between the clear small 4ths and the unclear large 4ths and no overlap between the clear large 5ths and unclear small 5ths; on the other hand, in the Mesopotamian formulation, the unclear large 4ths and unclear small 5ths are simply designated unclear in contrast to the surrounding clear 4ths and 5ths. In other words, hearing the small 4ths as smaller than the large 4ths and small 5ths or hearing the large 4ths and small 5ths as smaller than the large 5ths would satisfy the tuning structure that the Mesopotamian sources specified (just as in Pythagorean, equally tempered, and mean-tone tunings) intervals a)

letter name:	e	d	c	b	a	g	f	e_1
cents value:								
4th=480, 5th=720	480	240	0	0	-240	-480	-720	-720
4th=514, 5th=686	514	343	171	0	-171	-343	-514	-686

b)

interval	**letter-name instances**	**range of cents values (exclusive)**
small 2nds	e-f b-c	(0–171)
large 2nds	f-g g-a a-b c-d d-e	(171–240)
small 3rds	e-g a-c b-d d-f	(240–343)
large 3rds	f-a g-b c-e	(343–480)
small 4ths	e-a g-c a-d b-d c-f d-g	(480–514)
large 4ths	f-b	***(514–720)***
small 5ths	b-f	***(480–686)***
large 5ths	e-b f-c g-d a-e c-g d-a	(686–720)
small 6ths	e-c a-f b-g	(720–857)
large 6ths	f-d g-e c-a d-b	(857–960)
small 7ths	e-d g-f a-g b-a d-c	(960–1029)
large 7ths	f-e c-b	(1029–1200)

Figure 6. *Nīd(i) qablim* tuning based on extreme cents values for the clear 4ths and 5ths: a) individual tones, starting from b = 0 cents; b) ranges of values for each kind of interval (parentheses emphasize that these are closed values: e.g., a small 2nd is larger than 0 cents and smaller than 171 cents).

of 612, 600, and 579.5 cents would satisfy the mathematical models of the augmented 4th and, respectively, 588, 600, and 620.5 cents would satisfy the mathematical models of the diminished 5th, without confusion concerning distinctions among perfect 4ths, augmented 4ths, diminished 5ths, or perfect 5ths.

With regard to perception, Blackwood (1985, 199) has said that "a difference of 25 cents is sufficient … to identify a major [i.e., diatonic] scale as such." Here, Blackwood (1985, 199) is referring to differences between a minor 2nd and a major 2nd, between a major 2nd and minor 3rd, etc. As Blackwood (1985, 199, Table 73) shows, there is a single difference of at least 25 cents between any minor 2nd and any major 2nd, between any major 2nd and any minor 3rd, etc., if, relative to an octave of 1200 cents, the perfect 5th's size is between 689.286 and 715 cents, inclusive (and the perfect 4th's size is between 510.714 and 485 cents). However, Blackwood (1985) has not specified whether such a difference is necessary, rather than merely sufficient, and has not supported this claim concerning the "recognizability" of a diatonic scale with results of tuning measurements or psychoacoustical experiments. Instead, Blackwood (1985) has cited only his own "experience." Nonetheless, if Blackwood's (1985) experience is valid—that is, common to people in general—it would reduce the range of sizes for the clear intervals of Mesopotamia by about a quarter: (510.714-485)/(514.286-480)=~0.75.

ARISTOXENUS'S PERCEPTUALLY BASED APPROACH TO TUNING

Instead of being based on abstract fundamental-frequency ratios, the Mesopotamian formulation resembles Aristoxenus's perceptual formulation of ancient Greek tuning. Indeed, Aristoxenus's discussion is not merely perceptual by implication, for he was very explicit on this point, rejecting ratios directly:

> We try to give these matters demonstrations which conform to the *appearances,* not in the manner of our predecessors, some of whom used arguments quite extraneous to the subject, dismissing perception as inaccurate and inventing theoretical explanations, and saying that it is in ratios of numbers and relative speeds that the high and the low come about. Their accounts are altogether extraneous, and totally in conflict with the *appearances* (Barker 1989, 149, my emphasis).

The Mesopotamian sources also resemble Aristoxenus's treatment in the way they treat certain kinds of intervals. In Aristoxenus's view (Barker 1989, 139, 159), there are two kinds of intervals: concords and discords. Among the concords are intervals that today would be termed perfect 4ths, perfect 5ths, and perfect 8ves, as well as their supplementary counterparts: perfect 11ths, perfect 12ths, perfect 15ths, and so forth. Among these, Aristoxenus regards the perfect octave as privileged, for, as he puts it, any concord (e.g., a perfect 4th, a perfect 5th, or a perfect 8ve) whose size is increased by the size of a perfect 8ve is also a concord (e.g., a perfect 11th, a perfect 12th, or a perfect 15th). In contrast, this is not necessarily true if a concord's size is increased by the size of a perfect 4th or a perfect 5th—for example, two perfect 4ths result in a minor 7th and two perfect 5ths result in a major 9th, and neither of these larger intervals is a concord (Barker 1989, 139, 160). Aristoxenus considers this feature "a quality intrinsic and peculiar to the octave" (Barker 1989, 160).

Most important, for Aristoxenus the concordant quality of perfect intervals is regarded as perceptually given; that is, being heard as a concord is a "universal" in the philosophical sense. In the present instance, it is a feature shared by all particular perfect 4ths, perfect 5ths, perfect 8ves, and their octave supplements and absent from all other pairs of tones. Within the class or category of concords, Aristoxenus identifies perfect 4ths as the smallest (Barker 1989, 139, 160) and perfect 8ves by their role in generating concords from concords and discords from discords. In Mesopotamian theory, the counterpart of Aristoxenus's octave is merely implicit. As Anne Draffkorn Kilmer (2000, 114) has noted, there was no term for "octave" in Mesopotamian languages. Instead, we know of its function solely through the tuning sources' specification that strings 1 and 8 were jointly tightened or loosened.

In contrast to the Mesopotamian counterpart to the octave, the Mesopotamian tuning sources explicitly specified that intervals spanning four or five strings were either clear or unclear. For the Mesopotamian tuning sources, being heard as clear was a universal, just as being heard as a concord was a universal for Aristoxenus. In both formulations the principal distinction was between what would now be termed octaves on one hand and 4ths and 5ths on the other. Further, in Aristoxenus's theory, perfect 4ths and perfect 5ths are principally instances of a single kind of thing, namely, concords that are not octaves. Similarly, in the Mesopotamian tuning sources, the counterparts of these intervals are treated as a single kind of thing, namely, intervals that are either clear or unclear.

In the modern formulation of post-tonal theory, which is explicitly abstract, numerical, and mathematical (e.g., Rahn 1980), pitch classes are a counterpart to the perceptual octaves in Mesopotamian and Aristoxenian theory. As well, the single group that Aristoxenus's perceptually perfect 4ths and perceptually perfect 5ths comprise, and the single group consisting of Mesopotamian intervals that span four or five strings and are perceptually clear, correspond to post-tonal theory's single class of 5- and 7-semitone intervals, including their mod-12 equivalents. Further, whereas Aristoxenus seems not to have identified diminished or augmented 4ths or 5ths as special cases, his concept of magnitude would have distinguished them from their perfect counterparts. In contrast, Mesopotamian theory implicitly grouped all four- and five-string intervals and distinguished individual instances on the basis of whether they were perceptually clear or unclear. In further contrast, post-tonal theory has distinguished 5-semitone intervals from 7-semitone intervals in terms of the way in which their constituent pitches are ordered abstractly or registrally—that is, in terms of higher and lower, or in terms of their clockwise distance or angle of rotation on a one-dimensional circle or helix.

The manner in which Aristoxenus tuned a tense-diatonic tetrachord and, by immediate extension, a tense-diatonic octave (or more properly, a disjunct pair of tense-diatonic tetrachords) can be understood as a further parallel between his perceptual formulation of tuning and the Mesopotamian accounts. For Aristoxenus, a tense-diatonic tetrachord (e.g., e-d-c-b) consists of two whole tones and a semitone. A whole tone is the difference between a perfect 4th and a perfect 5th (Barker 1989, 140, 160). For example, b up a perfect 4th to e and e down a perfect 5th to a result in the whole tone b-a; a up a perfect 4th to d results in the whole tone e-d; d down a perfect 5th to g results in the whole tone a-g; g up a perfect 4th to c results in the whole tone d-c; c down a perfect 5th to f results in the whole tone g-f (Barker 1989, 142, 160, 168). Since Aristoxenus considers a semitone to be the difference between a perfect 4th and two whole tones (i.e., a ditone), the succession b-e-a-d-g-c-f results in the semitone c-b and the tense-diatonic tetrachord e d c b. Finally, if this succession is extended to e, a perfect 5th below b, the semitone f-e, results as well as the tense-diatonic tetrachord a-g-f-e, that is a whole tone below the tense-diatonic tetrachord e d c b with which it constitutes a disjunct pair of tense-diatonic tetrachords—that is, a tense-diatonic octave (Barker 1989, 167).

Aristoxenus quite emphatically insists that the tunings he prescribes were to be realized by ear rather than by means of instruments, especially the *aulos* (Barker 1989, 157–58). To be sure, without such a fixed-frequency instrument as a lyre to retain recently tuned fundamental frequencies, practical applications of his formulations for soft-diatonic, chromatic and enharmonic tunings would place relatively high demands on what in modern psychology is referred to as working memory (Cowan 2008). However, his tense-diatonic tuning would be cognitively much less demanding, requiring as it does only an incremental unfolding of perceptually perfect 4ths and perfect 5ths. Indeed, Aristoxenus identifies diatonic tuning as the "first and oldest" since "human nature comes upon it first," and in contrast, enharmonic tuning is for him the "most sophisticated, since perception becomes accustomed to it last, with difficulty, and through much hard work" (Barker 1989, 139).

Whereas Aristoxenus stresses that his tunings are to be realized by ear and refers to singing throughout the *Harmonic Elements*, his terms for individual tunings and tones suggest a lyre as their origin. For instance, the second tone from the top of his tetrachords is termed "forefinger" (*lichanos*). Commonly translated as, on one hand, "tense" or "intense" and, on the other hand, as "soft" or "mild," the difference between Aristoxenus's two kinds of diatonic tetrachord consists only in the second degree from the top being heard as, respectively, higher and lower, relative to the surrounding tones. Similarly, among Aristoxenus's three chromatic tetrachords, the second and third tones from the top of the tetrachord are lowest in his soft chromatic tuning (Barker 1989, 164–66). In sum, then, Aristoxenus's tense-soft contrast would be realized on a lyre by strings that were relatively tight and loose—that is, that had been relatively tightened and loosened, as in the Mesopotamian dichotomy. Indeed, the contrast he advances between tones that are vocally tensed and relaxed (e.g, Barker 1989, 142) might well be understood as having applied to the voice a distinction that was originally attributed to strings.

That Aristoxenus's formulation is fundamentally auditory contrasts with later writers' employment of the monochord, which relies on a mechanical application of ratios. In a monochord realization, the sizes of perfect Pythagorean 4ths and 5ths, relative to an octave of 1200 cents, would be, to the nearest cent, 498 and 702 cents. Whereas the ranges of 480 to 514 cents and 686 to 720 cents discussed above might seem rather large, it has been shown recently that perceptual constraints implicit in Aristoxenus's account of his hemiolic and soft chromatic tunings would ensure that the values of his perfect 4ths and 5ths would necessarily be much smaller in range, specifically, between 494 cents and 505 cents and between 695 and 706 cents, relative to an octave of 1200 cents (Rahn 2012, 4).

Such a range of perceptual values would correspond to only a third of the extent available to a diatonic tuning's generating intervals according to an abstract, mathematical formulation: (505-494)/(514-480) = ~32%, in contrast to Blackwood's (1985) range for the generators of "recognizably diatonic" tunings, which occupies fully 75% of the mathematically available extent. Indeed, Aristoxenus's demonstration that a perfect 4th comprises five semitones, which was subsequently controversial among theorists who formulated tunings in abstract, mathematical terms, would be arguably convincing if its thirteen perceptually perfect 4ths and 5ths were randomly distributed within this range, tending as they would to average ~500 cents. Conversely, Aristoxenus might have expected performers to realize perfect 4ths and 5ths within an even smaller range, for he claimed that concords "appear to have either no range of variation at all, being determined to a single magnitude, or else a range which is quite indiscernible" (Barker 1989, 168). Nonetheless, his insistence here is perceptual rather than abstract and mathematical, for concords "appear to have," that is, are heard as having, no range of variation at all, or else a range which is "quite," that is, not absolutely, indiscernible.

PEDAGOGY AND PRACTICE

As an important contribution to our understanding of the Mesopotamian tuning sources, Mirelman (2013, 49–53) supplements usual translations by adding the phrase "for me." As he explains in note 14 "for me" indicates "the existence of a particle in the Akkadian language which may indicate a directive action towards the speaker. This may indicate the pedagogic function of the tuning text." Recently, citing Åke W. Sjöberg (1974, 144, line 130), Mirelman (2010, 48n14) has pointed out that "[the] translation 'for me' is suggested, as the presence of a grammatical feature could be interpreted as such, especially in the light of the same grammatical feature in a literary text that discusses the place of music in the scribal curriculum. Here, the teacher asks his pupil whether he knows various instruments, including the *sammû* (i.e., harp or lyre) and the pupil reproaches his teacher, saying 'you do not say it to me.' "

Citing a conclusion recently advanced by Piotr Michalowski, Mirelman has noted further (2010, 46n6) that "[it] has also been suggested [i.e., by Michalowski 2010, 200–19] that although the roots of the tuning text(s) must lie in actual practice, these text(s) are not practical instructions for musicians, but rather scholastic exercises associated primarily with mathematical and lexical scribal practice."

Michalowski's distinction is between the training of apprentice professional musicians and the training of scribes. However, none of Michalowski's evidence counters the position that the Mesopotamian tunings were taught, rather than merely copied, and with a view to being put into practice by pupils who understood the terms employed. Indeed, none of Michalowski's evidence precludes the possibility that scribes and their students performed music according to the well-documented tuning prescription or that they transmitted the tuning formulation to other, less lettered persons. In any event, corroboration of the practical relevance of the tuning formulation is to be discerned in an aspect of an important Mesopotamian source that thus far has not been explained satisfactorily.

As Mirelman (2013, 44n2) points out, CBS 10996 is one of the sources that is indispensable for understanding not only the (re-)tuning sources but also the only pieces that survive in notated form. This is because CBS 10996 is the only source that links the names of dichords to particular pairs of strings. Indeed, it goes even further by providing for each dichord name not only a pair of terms that specify which pair of named strings among the first seven strings of a nine-string harp or lyre the dichord name refers to but also a pair of numerals from 1 to 7 that correspond to these strings. Further examination of CBS 10996 provides additional evidence for understanding Mesopotamian tuning as fundamentally framed in terms of seven degree classes and also suggests that CBS 10996's purpose was originally practical in general, and pedagogical in particular.

The numerals from 1 to 7 that CBS 10996 employs make it clear that the music it is concerned with is heptatonic—that is, the music is to be construed in terms of seven tones per octave, or as the present discussion maintains, seven degree classes per octave. As all three designations (i.e., the seven dichord names, the first seven string names, and the numerals from 1 to 7) are effectively equivalent and since the numerals from 1 to 7 clarify the idiom's heptatonic structure, I employ these numerals in the following discussion.

In CBS 10996, pairs of numerals are listed in the following order, with indentations added here for reasons that will become increasingly apparent (cf. also West 1994, 163):

1 & 5	
	7 & 5
2 & 6	
	1 & 6
3 & 7	
	2 & 7
4 & 1	
	1 & 3
5 & 2	
	2 & 4
6 & 3	
	3 & 5
7 & 4	
	4 & 6

Strongly suggesting a heptatonic framework, the initial numerals in the leftmost pairs proceed from 1 to 7 (i.e., 1, 2, 3, 4, 5, 6, 7). The second numeral is either 4 larger than the first (e.g., 1+4 = 5, 2+4 = 6, 3+4 = 7) or 3 smaller (e.g., 4-3 = 1, 5-3 = 2, 6-3 = 3, 7-3 = 4). These, then, are the 5ths and 4ths discussed above, and these 5ths and 4ths are arranged in stepwise order.

Mesopotamian specialists have not arrived at a consensus explanation of the rightmost pairs of numerals beyond noting that each of these pairs can be understood as comprising a 3rd or 6th. Moreover, the last four are not only arranged in stepwise order (i.e., 1 & 3, 2 & 4, 3 & 5, 4 & 6); as well, the first numeral of each is the same as the second numeral of the immediately preceding leftmost pair (i.e., 4 & 1 and 1 & 3, 5 & 2 and 2 & 4, 6 & 3 and 3 & 5, 7 & 4 and 4 & 6). But what sense is one to make of the initial three 3rd/6th intervals?

It has been pointed out that CBS 10996 merely says "string 1 *and* (*ù*) string 5," not "string 1 *then* string 5" (Rahn 2011a, 100). Accordingly, one can disregard, at least for the time being, the distinction between 3rds and their complements (i.e., their inversions relative to a 7-degree octave), namely, 6ths. Disregarding this distinction, one can discern a uniform pattern among the entire group of fourteen dichords as outlined in Example 1a.

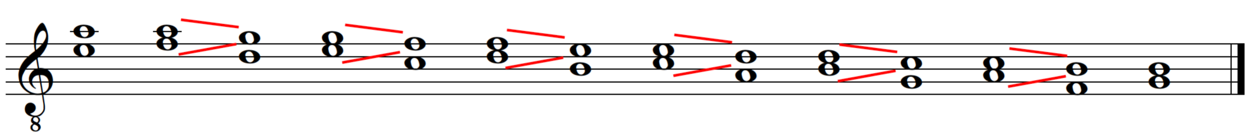
Example 1a shows that, in terms of degree classes, a single dichord progression proceeds stepwise throughout the entire succession of fourteen intervals. At the outset of this progression, the first and third dichords can be construed as a parallel-perfect-4th progression and the A-F and D-G dichords can be regarded as a stepwise contrary-motion progression from A and F to G combined with D of the E-A to D-G progression. Similarly for D-G to E-G to C-F, and so forth until the return to E-A that would complete the entire cycle.

Example 1b shows how the abstract, degree class structure outlined in Example 1a could be realized concretely by the first seven strings of a harp or lyre in *nīd(i) qablim* tuning. Example 1b also shows that within the constraints of seven strings (and irrespective of any of the seven tunings), a surface or foreground discontinuity would occur between the d-f dichord and the e-b dichord.

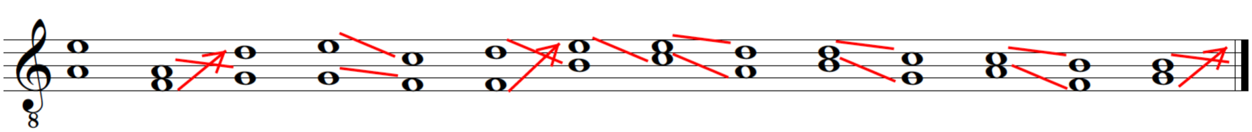
Another way of describing the cyclical progression of dichords is that in such a progression as A-F to D-G one of the string numbers increases by 1 mod-7 and the other string number increases by 2 mod-7. For example, in the progression A-F to D-G (i.e., 5-7 to 2-6), 5 (A) increases to 5+1 = 6 (G) and 7 (F) increases to 7+2 = 9 = 2 mod-7 (D). This is one of seven ways in which a 3rd or 6th can be followed by a 4th or 5th (e.g., A-F could be followed by E-A, D-G, C-F, B-E, A-D, G-D, or F-B).

Among the thirty-five notated Hurrian pieces (ca. 1350 BCE), there are fourteen progressions from a 3rd/6th to a 4th/5th (Rahn 2011a, 124, Fig. 14). Whereas one would expect

a) Stepwise succession of dichord pairs. Straight lines indicate "contrary-motion" melodic progressions down a step and up a step within dichord pairs.



b) Stepwise succession of dichord pairs within strings 1 to 7 of a harp or lyre. Arrows indicate "octave displacements" of "contrary motion" in down-a-step and up-a-step melodic progressions within dichord pairs.



Example 1. The fourteen dichords of CBS 10996.

only 14/7 = 2 of these to combine the up-a-step and up-two-steps melodic progressions, most do so—specifically, 9 of the 14, for which the binomial probability is 0.01; that is, the probability that this disproportionately frequent employment of the up-a-step/up-two-steps progression would have occurred by chance is only one in a hundred (Rahn 2011a, 128, Fig. 18). In short, the cyclical progression of dichords in CBS 10996 is not only highly unlikely in its own right, it occurs improbably often in the only notated Mesopotamian pieces that have survived. Accordingly, CBS 10996 can be regarded as an exercise in realizing the kind of 3rd/6th to 4th/5th progression that, as far as we know, was most idiomatic in Mesopotamian music.

As well, if one proceeds from the relatively abstract realm of pitch classes (or more properly, string classes) and performs the progression on only the first seven strings of a harp or lyre, one has to use all four registral orderings of the progression, so that a novice's fingers would get a thorough workout by playing the entire progression fluently: A-F to D-G requires a different shift in one's finger pattern than E-G to C-F; D-F to E-B differs from both of the preceding; E-C to D-A differs from all three of the preceding but is the same as the next two (D-B to C-G and C-A to B-F), and B-G to E-A has the same pattern as A-F to D-G, bringing the cycle back to the beginning (Example 1b). In this way, the entire dichord cycle would provide for a performer's fingers a workout that was not only idiomatic but also thorough in the large-motor movements it would demand.

Finally, if one restores the original ordering of the individual strings of each dichord by (over-)interpreting the connective "and" ('*ù*' in the original) as "then," one can discern within a realization by a nine-string harp or lyre a patterning within the entire fourteen-dichord cycle. In the initial three pairs of dichords, the first dichord proceeds upward by five strings (i.e., "down a 5th" in pitch) and the second tone of the second dichord is the same as the second tone of the first dichord, as follows (with underlining indicating the common tones):

e ***a*** f ***a***; d ***g*** e ***g***; c ***f*** d ***f***

In contrast, in the remaining four pairs of dichords, the first dichord proceeds downward by four strings (i.e., "up a 4th" in pitch) and the first tone of the second dichord is the same as the second tone of the first dichord:

b ***e*** ***e*** c; a ***d*** ***d*** b; g ***c*** ***c*** a; f ***b*** ***b*** g

Moreover, these dichord pairs are linked by patterned progressions from one dichord to the next. In the first group of "down a 5th" dichord pairs, the second, third, and fourth tones of each dichord and the first tone of the next dichord constitute the progression down a 3rd, up a 3rd, up a 4th, a fingering pattern that extends to the first tone of the "up a 4th" dichord pairs (again, indicated by underlining):

e a f a; d g e g; c f d f; (b)

By contrast, in the second group of "down a 4th" dichord pairs, the second, third, and fourth tones of each dichord and the first tone of the next dichord form the progression unison, down a 3rd, down a 3rd, this pattern coming full circle at the first tone of the entire progression's first dichord:

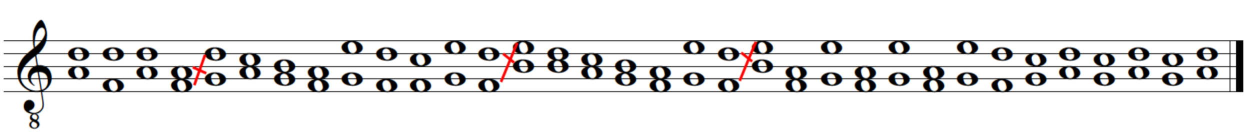
b e e c; a d d b; g c c a; f b b g; (e)

Just as those trained in the performance of European common-practice music play their chordal exercises in both "block" and "broken" form, the successive dichords of CBS 10996 can be understood as providing idiomatic exercise material that would facilitate fluency in both sets of skills. In this regard, Mirelman's (2013, 53) observation of the limited and minor hand movements involved in the Mesopotamian procedure for retuning a harp or lyre is even more significant, especially as retuning comprised common strings and adjacent strings, as did CBS 10996's pattern of 4th/5th and 3rd/6th intervals.

COMPOSITIONAL DESIGN

Additional aspects of CBS 10996 seem relevant not only to performance practice but also to compositional design. As noted above, the dichord pairs proceed upward with regard to string numbers and downward with regard to pitches and degree classes, a relationship that results from construing tightening as the topic of the first part of the central tuning manuscripts and loosening as the second part's topic, as Mirelman does (2013, 48–54) in identifying as the consensus understanding of pitches (and degree classes) in Mesopotamian tuning. Among the thirty-five notated pieces that survive, immediate progressions from a 3rd/6th interval to a 3rd/6th interval tend to proceed downward by a single degree class. In the only piece that does not survive in fragmentary form, namely, h.6, such progressions appear throughout the piece's middle (Example 2). Toward the piece's end, 3rd/6th intervals oscillate in immediate stepwise succession before resuming their stepwise descent.

Whereas the only progressions from a 4th/5th interval to a 4th/5th interval appear in the piece just described, one can note that these are employed differently in the piece's different passages. At the beginning of the piece, a 4th/5th dichord is repeated in non-immediate succession. In the middle of the piece, 4th/5th intervals proceed downward in non-immediate succession by a single degree class; thereupon, the last 4th/5th dichord of this non-immediate



Example 2. The thirty-four dichords of h.6 within strings 1 to 7 of a harp or lyre. Straight lines indicate melodic progressions down a step and down two steps within dichord pairs during the middle of the piece; note that these correspond to the "contrary-motion," down-a-step and up-a-step melodic progressions within dichord pairs in Exx. 1a and 1b.

downward succession is repeated at the unison. And at the end of the piece, 4th/5th intervals proceed in immediate succession, forming an oscillating stepwise passage that concludes at the dichord with which the piece began. Finally, the up-a-step/up-two-steps progression, which in the conventional view would be down-a-step/down-a-3rd in pitches or degree classes, is employed solely in the piece's middle.

To summarize, in the single piece that survives in non-fragmentary form, 3rd/6th intervals proceed immediately downward by step in the middle and oscillate immediately stepwise toward the end. Similarly, in the piece's middle, 4th/5th intervals proceed downward by step, albeit non-immediately, and oscillate immediately stepwise at the very end. At the beginning, the piece's first dichord, a 4th/5th dichord, is non-immediately repeated at the unison and this 4th/5th dichord is repeated at the very end of the piece. Similarly, toward the end of the piece's middle, the last dichord of the non-immediate downward stepwise succession of 4th/5th intervals is repeated. Progressions from 3rd/6th to a 4th/5th, proceeding according to the down-a-step/down-a-3rd pattern, occur in the piece's middle.

In short, the downward stepwise framework of 4th/5th intervals in CBS 10996 is characteristic of the middle of the only non-fragmentary piece and the concomitant downward-stepwise pattern of 3rd/6th intervals is characteristic of this piece's middle and of a tendency throughout all thirty-five Mesopotamian pieces that survive fully or in fragmentary form. From these observations, then, one might conclude that dichord progressions that have tended to survive in the fragmentary notations have been portions of their pieces' middles—a conclusion that seems consistent with the portions of clay that have remained intact (see Laroche 1955, 1968; Dietrich and Loretz 1975).

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