Preliminary Remarks on a Helical Representation of Musical Time

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In this article I describe a system for representing musical timing as a helix, enabling the three-dimensional visualization and printing of both micro- and piece-length temporality. Examples from the Balinese gamelan repertoire serve as the principal demonstrations. I describe the functionality of Mathematica code developed to produce the visualizations, providing publicly accessible links to the code, data, and output files. Rather than a study presenting extensive findings, this article outlines a method and describes preliminary examples. The model represented here aspires neither to a mathematical formulism—a computational approach to model composers' processes—nor to a cognitive psychology, which would attempt to reveal ostensibly universal mental representations of musical time. Instead, this study combines ethnography with music theory to suggest one possible musical imagination: the helix as a spatial heuristic for listening.

The concepts informing the present project emerged from interviews with Indonesian musicians, composers, and audiences conducted as part of an ethnographic study of Balinese contemporary music (McGraw 2013). Many young informants described a general qualitative divide between the performances of their forefathers and themselves. These differences were often ascribed to radical changes in lifestyle and culture; the former described as living in tightly knit agrarian villages, the latter—often conservatory-educated cosmopolitan composers living in peri-urban, multicultural housing developments—as experiencing more intensely the alienating forces of urban modernity. Many young musicians suggested that they "can no longer play like [their] grandfathers" as a result, in part, of a perceived loss of agrarian, premodern communitas. Several informants linked this loss to the influence of Western culture and temporal regimes, both in the rhythms of the nine-to-five workday and the solidly metronomic grooves of Western-style popular music. Such forces, many suggested, functioned to homogenize (with the implication of impoverishment) and simplify what was once a richer and more variegated ecology of musical styles in Bali. These styles are iconic of distinct communities; their perceived erosion betokened the loss of community itself.

When asked what—besides obvious differences of context and orchestration—might differentiate their musics from those of previous generations, young musicians and composers appealed to a vague difference of feeling (*rasa*). According to one composer: "I don't know what the difference is, but I can always feel it." Temporality often resides within an implicit realm of feeling in both Balinese and Western musical understanding. For many informants style was most powerfully embodied in an ensemble's unique structuring of musical time, how an ensemble shaped a work through locally distinctive temporal transformations. These transformations resided primarily at the micro (groove) and macro (piece-length) frames—those levels of temporality most resistant to representation and analysis within the technology of standard Western staff notation, calibrated for the "meso" frame of pulse and rhythm.

Many forms of Asian music have been caricatured as temporally cyclical and therefore non-processive or static. Certain music theorists and ethnomusicologists have suggested that Balinese music is primarily repetitive, cyclical, and isoperiodic, finding that it represents "nonlinear cultural attitudes and life styles" (Kramer 1988, 24), and that it has no climax. The lineage of this thought seems to have flowed thus: Kramer (1988), quoting Geertz (1973b) referencing McPhee (1966).

It is possibly due to romantic anthropological notions of Balinese culture as existing somehow out of time that theorists would be inspired to suggest that Balinese music is made up of rhythmic cycles which repeat "seemingly . . . without end" and that it is thus not surprising that Balinese musical performances "simply start and stop but have neither beginning gestures nor ultimate final cadences" (Kramer 1988, 24). Such statements play into colonial tropes of Oriental life as timeless and of Asian culture as non-processive and, by extension, European culture and thought as progress-based and teleological.

We might trace this idea back to Marx's notion of the Asian Mode of Production, which Anderson describes as a political history that was "essentially cyclical: it contained no dynamic of cumulative development. The result was the ". . . inertia and immutability of Asia" (Anderson 1974, 483). This image reappears in both Bateson's (1949) theorization of Balinese society and expression as a "steady state" and later characterizations of the Southeast Asian state as a static recapitulation of cosmological beliefs in which the *negara* state produces a magical correspondence between the universe and the world of men "acquiring a power with which to dominate space, time and social relations" (Heine-Geldern 1956, 1). It is seductive to think of Balinese music in terms that accord with an assumed Balinese worldview and concepts of time-in-general. Becker suggests that the most prominent feature of iconic power in gamelan music is coincidence: coincidences of interlocking cycles ranging from micro to macro levels, all iconic with the cycles of calendars and cosmos and thus, for the Javanese (and, one assumes, Balinese), appearing completely "natural" (Becker 1981, 207). Similarly, many scholars have connected Indian concepts of *tal* with the Hindu cosmological notion of *kalpa*, or world cycles. In classical Indian cosmology the *kalpa* repeat, but not in detail, just as do *tala* cycles. However, as Clayton points out, "Musical time is no more a circle, or a wheel . . . than it is a rule or tape measure . . . or indeed a helix or a wave" (Clayton 2000, 19).

Although his transcriptions mask temporal complexity, McPhee was aware of the temporal richness of Balinese gamelan:

> In conception Balinese music is static, whereas ours is dynamic and generally the expression of a crisis, a conflict. In execution Balinese music is extremely dynamic, while paradoxically much of our own music, especially that of the nineteenth century, seems by comparison, turgid and lethargic (1935, 164).

Although it is difficult to maintain that Balinese music—indeed *any* music—is structurally static, performances of even the simplest, most repetitive, and "cyclic" of traditional works, such *Bapang Selisir* described below, demonstrate a dynamic and linear narrative of temporal transformation. An aim of the visualization method described below is to demonstrate this continuous narrative of temporal transformation without ignoring repetition (as in McGraw 2008). Much of Balinese gamelan music is indeed cyclic, incorporating the repetition of metered forms that feel similar despite continuous temporal transformation (pushes and pulls, changes in groove, switches to double time, etc.). While a critique of the philosophical/ethical underpinnings and effects of the characterization of "Asian music" as cyclical is important and needed, the entire project presented in this paper depends on the reality of isoperiodic equivalence. In human expression and experience, the tension between cyclicity and linearity may be as irreducible as that between nature and nurture, being and environment. I argue that helical representations allow us to see temporal identity and difference simultaneously and in a more balanced manner. However, as I explain below, two-dimensional representations may be preferable when it is important to perceive all data points at once or when our gestural interpretations of musical sound are primarily two-dimensional, as in the vertical spatialization of pitch.

A GAMELOMETER

A multitrack, sensor-based recording system was developed to investigate informants' descriptions of stylistic difference between regional ensembles (see Photo 1).[1] Individual piezo sensors running into a 24-track field recorder were placed on gamelan keys, chimes, and drums, allowing for complete isolation of each keystroke and facilitating the automated detection of onsets in analysis programs such as Sonic Visualizer. Recordings were made in March 2011 in five villages known island-wide as possessing historically iconic ensembles that embodied local style and identity: Jagaraga, Perean, Pinda, Gladag, and Pengosekan. The gamelan from Pengosekan, the renowned Cudamani ensemble, represented the youngest group of musicians, populated by many of the contemporary composers who described the changes in style described above.

Photo 1. A sensor-based recording system, designed by Joel Mellin

[1] The New York-based composer and electronic engineer Joel Mellin designed and built this system and travelled to Bali to assist in recording. For a full description of the recording system and schematics, see http://www.joelmellin.com/gamelometer/

THE STRUCTURE OF TIME IN BALINESE GAMELAN MUSIC

The structure of traditional Balinese gamelan music is overwhelmingly binary. Low gongs mark the overall form or its phrase subdivisions; smaller gongs (*kempur, klentong*, *kempli*) mark subdivisions—usually binary—within the gong form (*gongan*). Melodic orchestration is also typically binary and hierarchical. The *ugal* metallophone or *trompong* gong-chime performs semi-improvised, rhythmically dynamic melodies (*lagu, neliti*) oriented around the quarter- and eighth-note density level. These are abstracted at the quarter note on alto-range *penyacah*, at the half note on tenor-range *calung,* and at the whole note on the bass *jegogan*. The two-octave *gangsa* metallophones, one and two octaves above the *penyacah,* perform complex interlocking figurations known as *kotekan*. These are often divided into two rhythmically syncopated parts (*polos* and *sangsih* respectively), which elaborate the contour of the *neliti*, the principal melody. This framework, however, is only theoretical; plentiful examples can be identified in which this abstract hierarchy is dissolved by compositional and stylistic prerogatives (see Tenzer 2000).

The analysis of pulse in most Balinese gamelan music is comparatively transparent, as it is consistently and unambiguously performed on the *kajar*, a small timekeeping horizontal gong-chime. That is, the pulse in Balinese music is phenomenal; all listeners and musicians are in agreement as to where it lies and can literally point to it. Similarly, larger-scale metrical structures are also phenomenal and unambiguous in Balinese music, unlike meter in much Western art music, which is primarily conceptual; or in West African music, which can be ambiguous or variously interpreted. In Balinese music there is little room for what Locke (1998, 22) calls the “gestalt flip” potential in the experience of listening and performing Ghanaian Ewe drumming.

Pulse in Balinese music is punctual rather than durational; the *kajar* is damped by the left hand while being struck by a mallet held in the right, producing a staccato "tuk" sound. In most contexts the *kajar* performs every four notes relative to the fastest, densest rhythmic layer of the gamelan, most often played by the *gangsa* metallophones. Western theorists typically notate the *kajar* at the level of the quarter note. However, in certain textures, such as the slow sections (*pengawak*) of *pelegongan, semar pegulingan,* and *gong kebyar* repertoire, the *kajar* plays once every eight *gangsa* notes.

THE EXAMPLE: *BAPANG SELISIR*

To evaluate informants' descriptions of the localized structuring of musical time as an embodiment of regional style, a simple and short work shared between the regional ensembles was chosen as the first object of analysis. *Bapang Selisir* employs an eight-beat gong pattern (the *bapang*), outlined by the low gong on the eighth pulse and bisected by the high *klentong* gong on the fourth pulse.[2] It is performed in the *selisir* mode, most often represented in standard Western notation as C♯, D, E, G♯, A; or in Javanese and Balinese cipher notation as pitches 1, 2, 3, 5, 6. To some informants *Bapang Selisir* hardly amounted to a "composition" (*komposisi*), but was a simple "pattern" (*pola*) used to accompany female dance styles or stock characters in dance-drama. It emerged from the fog of prehistory, a traditional work known to all but with no known composer. For the purposes of this analysis (and because of the limited number of sensors), only the onsets performed on the *calung, gong, kajar,* and interlocking *gangsa* are analyzed below.

[2] Balinese meter is end-weighted; colotomic points are understood to land on beats 4, 8, 16, 32, etc.

Example 1. *Bapang Selisir,* Western notation,
large gong indicated by G, *klentong* by +

Example 2. *Bapang Selisir,* Balinese *aksara* notation,
gong indicated by circle; note the end-weighted phrasing

As seen in Examples 1 and 2, both Western and Balinese notations are calibrated to represent the temporal frame of the bar or phrase. Neither is well suited to rigorous descriptive representations of the temporal flux (either macro or micro) of actual performances; the Balinese notation is primarily mnemonic, not even indicating the method of interlocking to be incorporated. Both would seem to suggest the "static" cyclicity predicted by the orientalist descriptions of Asian music described above.

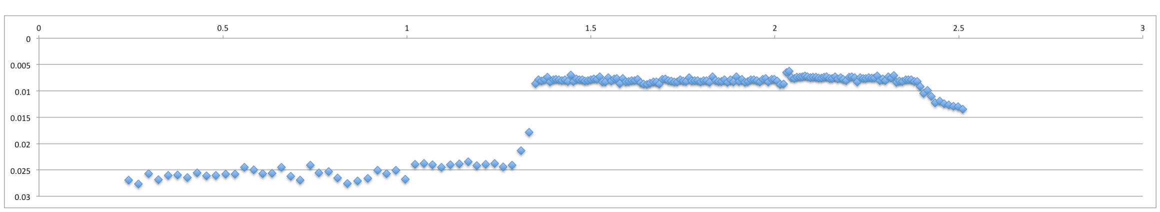
DIMENSIONS OF TEMPORAL REPRESENTATION

In the interest of space I will limit the present analysis to performances of *Bapang Selisir* from the villages of Pinda, Perean, and Pengosekan, focusing first on the Pinda performance (click here to download a recording). How can we best visualize the ways in which the Pinda musicians structure time in their performance? The one-dimensional, "smooth" representation of the *kajar* onsets shown in Example 3 provides us with a richer, more complex depiction of tempo than that allowed by the standard notations above but conveys a completely linear sense of transformation, providing scant information about form. We can see, however, that the musicians appear to switch to a much faster tempo around the middle and to slow towards the ending.

Example 4 gives a two-dimensional representation of the same data, in which the *y* axis indicates beats per minute (tempo, rising with *y*), and the *x* axis represents track timing. This representation makes more apparent the flux within what appeared to be comparatively stable sections in the 1D representation and allows us to formulate a clearer sense of structure and comparative tempos.

Example 3. One-dimensional representation of *kajar* onsets,
Bapang Selisir, Pinda village

Example 4. Two-dimensional representation of *kajar* onsets,
Bapang Selisir, Pinda village



“FIE, FIE, HOW FRANTICLY I SQUARE MY TALK!”

Two-dimensional examples such as those above have been widely used in the temporal analysis of music. They are comparatively simple to generate, either with automated detection of recorded onsets or through the “tempo tap” features in programs such as Sonic Visualizer. They are well suited to the technologies of our discipline, such as books, paper, and static images on websites. However, as new technologies—such as 3D vector graphics and printing—become more widely distributed, understood, and user friendly, we may pause to consider the ways in which our core representational technologies, essentially unchanged for centuries, have encouraged us to “square” our analyses to fit within them, and consider the possibilities and problems of new modes of representation.

Three-dimensional graphics programs have made possible the visualization of highly complex concepts such as hyperdimensional representations of tonal and atonal repertoires (see Callender, Quinn, and Tymoczko, 2008; Tymoczko, 2011). Hyperdeminsional analyses are a provocative intellectual challenge that might prove revolutionary for the development of new musical imaginations. However, Balinese musicians typically describe their music in terms of the four-dimensional world of human embodied experience. Zbikowski (2005) argues that, across cultures, the experience of music is deeply inflected by processes of cross-domain mapping through which musical gestures may be perceived as iconic of bodies moving through four-dimensional space-time. The description of music as a wave, a revolution, a straightaway, or a slope that I heard in my ethnographic encounters supports this and is the principal reason I restrain myself to three-dimensional representations here.

A HELICAL REPRESENTATION OF MUSICAL TIME

Previous authors, including Pressing (1993), Chaudhary (1997), and Clayton (2000), have discussed helical models for musical timing.[3] Rigorous development of a helical model, however, has been impractical because of the technical limitations of our overwhelmingly 2D discourse. Here I demonstrate that its continued development is now possible and practical, allowing for the creation of interactive 3D graphics and the printing of objects to represent musical time.

The Formula

A simple trigonometric formula was used to generate the helical model in the examples described below. Let c be the onset timing of an instrument performing a periodic role; in the current examples this may be the *kajar, calung,* or gong. The derivative c' is the timing difference between c_x and c_{x+1} (the subsequent onset timing). Let n be the number of onsets per cycle performed by the instrument represented by c. In our case, this would typically be 1 for gong, 4 for *calung* and 8 for the *kajar.* Let t be the onset number of instrument c (1st onset, 2nd onset, etc.). For the three dimensions of x, y, and z we use:

x = c'[t]*Cosine[2 π*t/n],
y = c'[t]*Sine[2 π*t/n],
z = c[t]

Here inter-onset timing is represented by the radial component defined by the x and y coordinates. The longer the inter-onset timing (the slower the tempo), the longer the radius length from origin. A shorter radius indicates a faster tempo. The radius is redrawn

[3] See also Shepard (1982) and Chew (2007) for applications of helical models to pitch space.

n times to corresponding positions on the circle. If *c* represents the *calung* performed 4 times per gong ($n = 4$) then the radius is redrawn 4 times equidistantly to corresponding positions on the circle. Absolute track timing is represented along the *z* axis, or longitudinal component, pushing the circle forward each time the radius is redrawn. Developing the formula in Mathematica, an interpolation function allows us to draw a smooth parametric curve between each of the onset points of an instrument whose timing is given by *c*. An inverse function allows us to string along the onsets of any other instruments onto the helical curve of the instrument represented by *c,* indicating both their exact relationship in track time and their relative relationship to the timing of instrument *c.* Scaling constants *s* and *k* allow us to shape the resultant helix for easy viewing and manipulation and for practical 3D printing. *K* represents the longitudinal (*z*) distance between cycles and should be small enough to allow us to see correspondences between adjacent cycles. *S* represents a constant for scaling the radial component. The full computational function for drawing the helix is:

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helix[t_, k_, n_, s_] = {s*c'[t]*Cos[2 π*t/n], s*c'[t]*Sin[2 π*t/n], k*c[t]}
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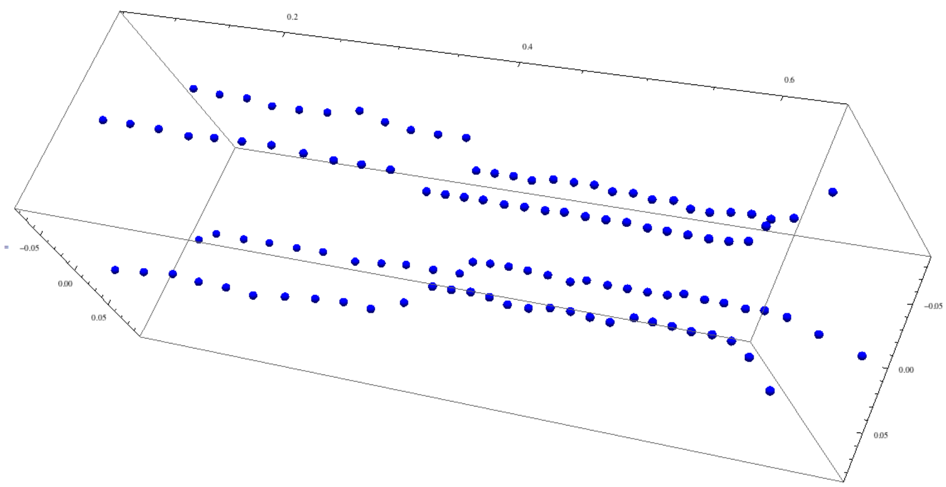
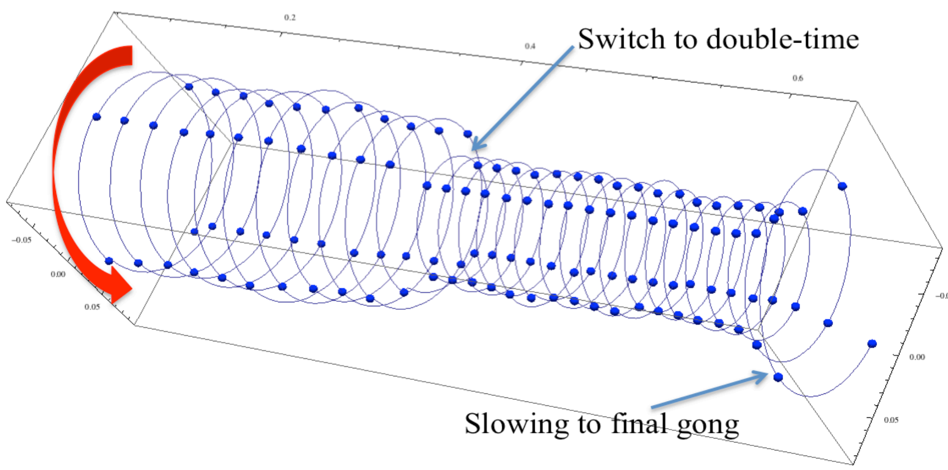
Looking back at the *Bapang Selisir* example from Pinda village, entering the *calung* onsets for *c* (which the Pinda musicians perform 4 times per gong) while scaling *k* at 0.25 and *s* at 3 produces the object in Example 5. Drawing the interpolated parametric helical curve, as in Example 6, allows us to visualize more clearly the cyclical structure. Now we are better able to see the continuous ebb and flow of the *calung* onsets (flowing clockwise with the red arrow) in combination with the cyclical structure of the form

Example 5. Helical representation of *Bapang Selisir*, Pinda village, read left to right; *calung* onsets only, $k = 0.25$, $s = 3$



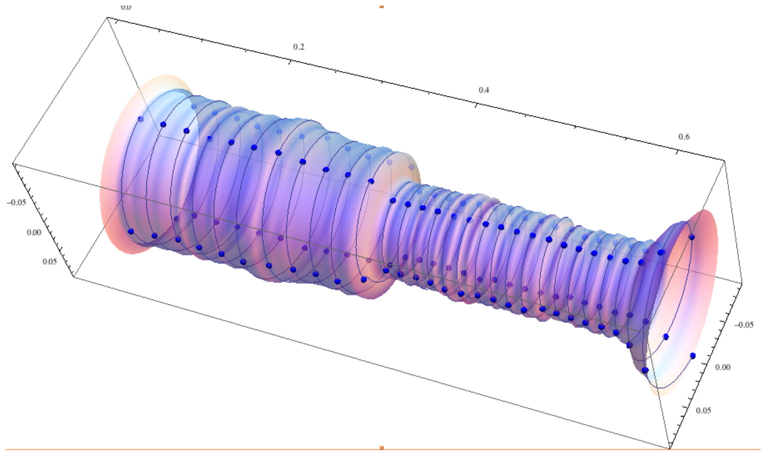
Example 6. Helical representation of *Bapang Selisir*, Pinda village; *calung* onsets with interpolated curve



itself.[4] We see that the form opens in a slow, although not metronomic, tempo and then around the middle of the performance breaks into double time, again far from metronomic,

[4] It is vital to remember that for the purposes of this paper we are in the awkward position of flattening into 2D what are in reality interactive 3D objects that in the development environment can be easily turned, resized, rotated, and passed through. Click here for a demonstration of the ways in which the models can be manipulated in a 3D environment.

Example 7. Helical representation of *Bapang Selisir*, Pinda village; *calung* onsets with interpolated curve and surface



before slowing towards the end through the final two cycles.

Drawing a partially opaque surface, as in Example 7, reveals better the ripples in tempo and allows us to focus on and compare particular quadrants of the helix.We now have a watertight structure that can be printed as either a solid object or a shell on a 3D printer. Furthermore, the surface allows us to easily see the structural gestures that the Pinda musicians incorporate to imbue this simplest of "works" with the enlivening tension and dynamism between linear transformation (change) and a sense of return (cyclicity). In Example 8 we see the temporal compressions leading to *angsels* (the stylized "hiccups" in the texture coordinated with dance moves), the temporal relaxation following some *angsels,* the sudden temporal compression following other *angsels* (*ngeseh*), the dramatic and sudden switch to double time in the middle, and the exponential rallentando leading to the final gong.[5]

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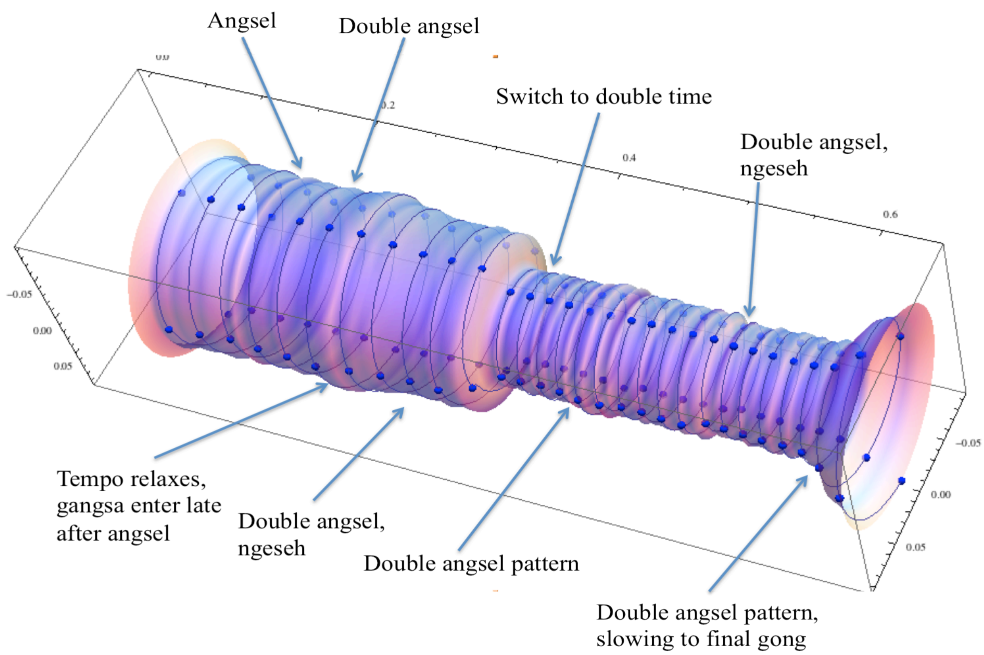
[5] It is noteworthy that many of the Balinese musicians who have seen these models are able to make immediate sense of them without prior explanation (or even axes). Some are even able to discern one village performance from another based upon the shape of the helix. Several noted the iconicity between

Example 8. Helical representation of *Bapang Selisir*, Pinda village, formal gestures revealed by surface distortions

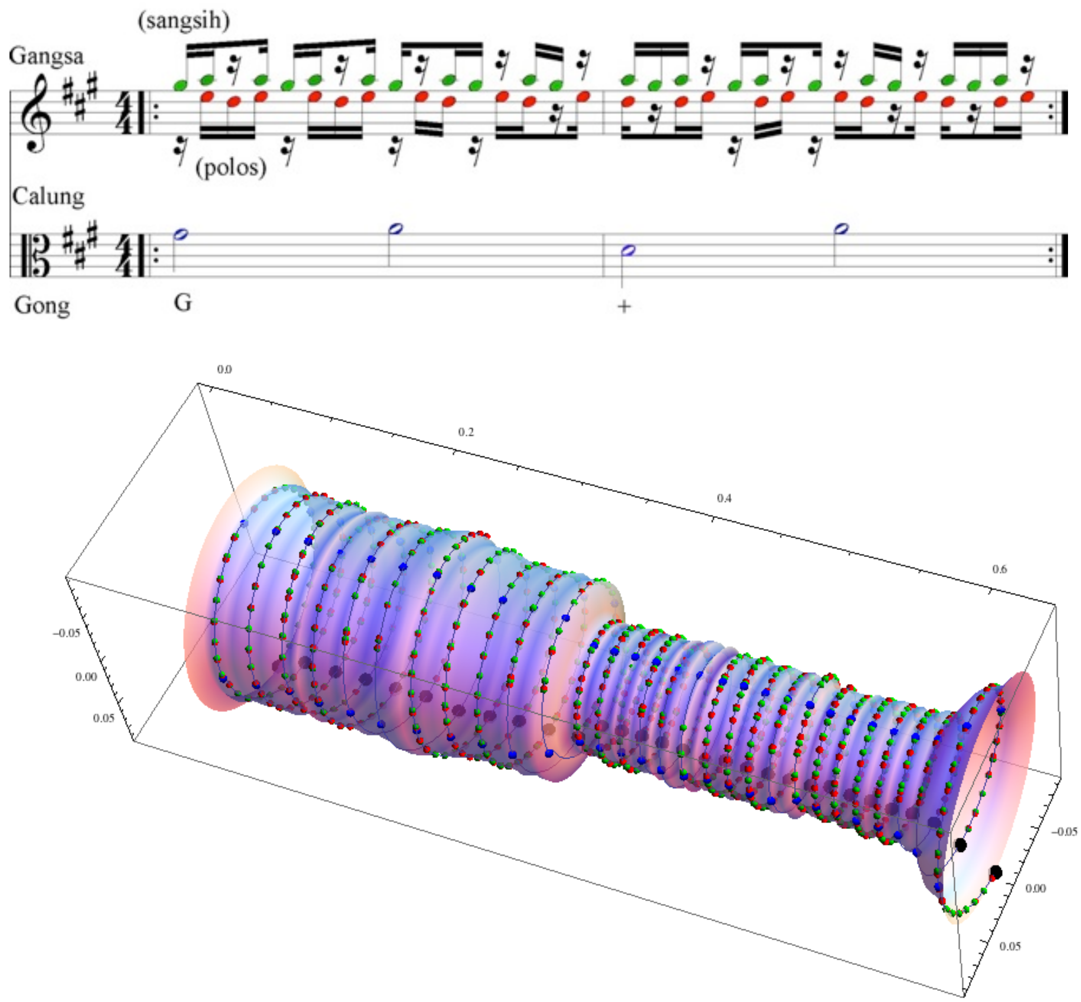


To make the relationship between the helix, instrumental onsets, and the music itself even clearer, code was developed to animate a "bouncing ball" to traverse the helix in synchrony with an embedded room recording of the entire ensemble. Click here to download (requires Wolfram's free CDF player: http://www.wolfram.com/cdf-player/).

Using an inverse function we can now string along the onset data from other instruments and compare them to their representation in standard Western notation. Example 9 illustrates two periodic instruments, *calung* (blue) and gong (black), as well as the onsets of the *gangsa,* both *polos* (red) and *sangsih* (green) parts. While appearing visually dense when flattened into a 2D representation, the onsets and their interrelationships become clearer when manipulated in the 3D environment or when printed as an object.

the Pinda performance and the large prehistoric bronze drums that many scholars speculate represent the historic progenitor of the gamelan.

Example 9. Helical representation of *Bapang Selisir*, Pinda village, including gong (black), *calung* (blue), and *gangsa* (red, green) onsets; *calung* radius, $k = 0.25$, $s = 3$

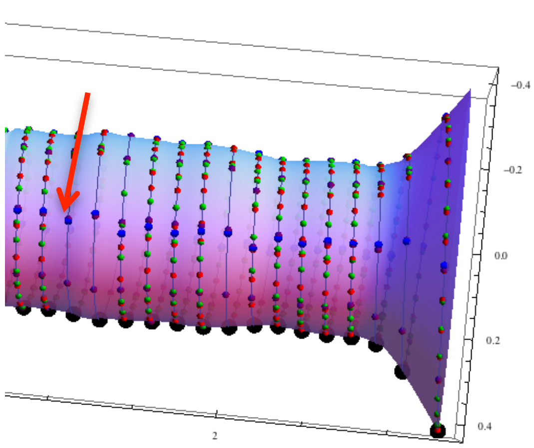


OPTIONS FOR THE RADIAL COMPONENT

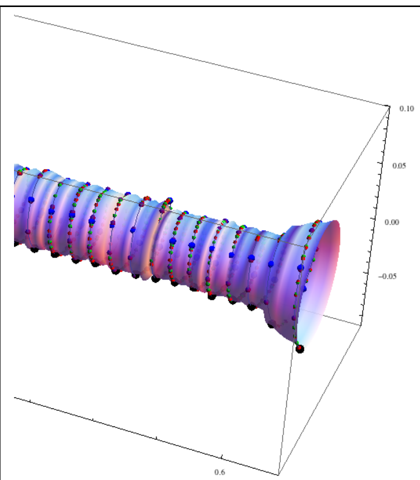
Choosing a different instrument for *c*, with a different *n* (number of onsets per cycle), produces differently textured surfaces and helices of the same performance. Assigning *c* to the gong, for which the radius is redrawn only once every cycle, produces a smoother surface than would be the case when choosing the *kajar* for which the radius is redrawn eight times per cycle, thus reflecting the more subtle timing discrepancies between each beat, as seen in Example 10.

A higher *n* results in a more finely textured surface. The onsets for the radial instrument (*c*) will appear aligned along the longitudinal (*z*) axis and all other onsets are drawn in reference to it. Performances viewed from the framing perspective of the gong appear as an aerial photograph, revealing broad outlines and allowing the comparison of relative onset timings of all instruments between adjacent periods. We see that the third (penultimate) *calung* tone in the third cycle, indicated by the red arrow in Example 10, appears proportionally later in the cycle as compared to structurally equivalent *calung* tones (the third in the cycle) of the previous two gong cycles. Performances viewed from a faster moving periodic layer, such as the *kajar,* are closer to the "ground" of the musical time. Any periodic onset in an expressive human performance is a point in a slope of change. This slope appears differently contoured when viewed from different hierarchical levels. The tempo marking indicated by any periodic point does not suggest internal regularity of the periodic activity within it; an irregular *kajar* pulse can be performed within otherwise equidistant gong cycles (*gongan* of comparable clock time).

Example 10. Comparing different instruments for *c*; excerpt, *Bapang Selisir,* Pinda village; *kajar* onsets are purple



Gong radius: $n = 1$, $k = 1$.



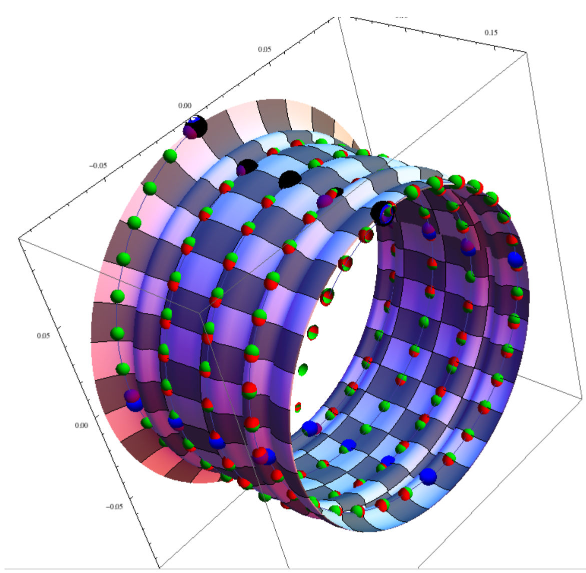
Kajar radius: $n = 8$, $k = 0.25$.

THE GRID

While the helix and the surface drawn upon it allow us to view both the cyclic and narrative structure of piece-length temporality, drawing a grid upon the surface allows us to investigate microtiming. In Example 11 the helix is drawn from the *calung* perspective; a grid divides each cycle into 32 divisions, drawn equidistantly between each *calung* onset and indicated by alternating grey and white bars.

Balinese musicians often speak of gong forms (such as the *bapang*) as if they were volumes into which the finer notes of the *gangsa* are poured, implicating an equidistant, binary structuring of time all the way down. Compositionally, form in traditional contexts is often determined top down before its contents are composed. However, temporal transformations emerge bottom up, with the fast rhythmic interaction of non-colotomic instruments (*gangsa, kendang, reyong, ceng-ceng*) guiding the placement of periodic gongs (*jegogan, calung* and *kajar*). Rather than speaking of *gangsa* onsets as being late or ahead relative to periodic onsets and the binary subdivisions they imply, we could argue

Example 11. Drawing a grid to visualize microtiming; beginning, *Bapang Selisir,* Pinda village



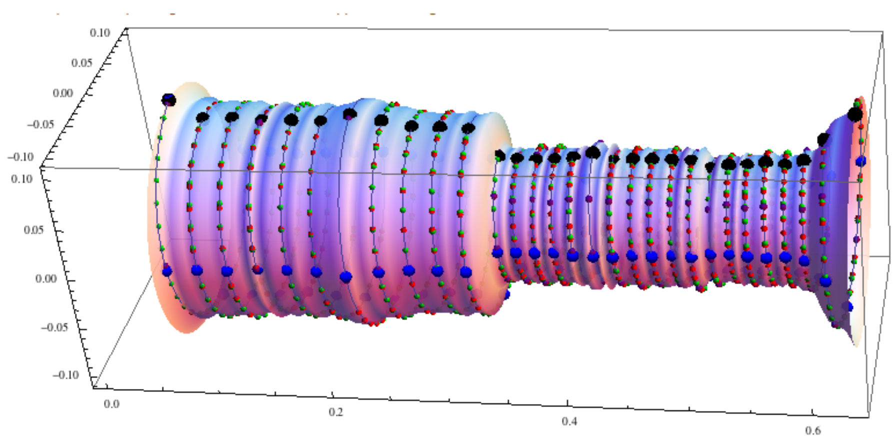
that the periodic instruments themselves are late, on, or behind the placements suggested by the synthesis of the fastest subdivisions. However, *kajar* players may disagree and it is often the case that the onsets of fast-moving instruments deviate (sometimes quite dramatically, in the case of *kendang* drumming cues) from the equidistant, binary subdivisions (sixteenth notes in the current example) implied by both the binary structure of Balinese music and standard Western notation.

Benadon (2007, par. 1), citing the evidence reviewed by Clarke (1999), argues that "musicians often place attack points along a continuum of beat subdivision values rather than on the predetermined slots afforded by metrical grid spacing, thus imbuing the performance with expressive depth." The onsets performed by *gangsa* performers often align with Benadon's (2009, 136) description of "gridless beats": "a beat containing onsets that are not aligned with its isochronous subdivisions." Traditional Balinese gamelan music often appears to be ordered in a rigorously binary manner—a straightforward binary splicing from the *gong* to the tactus—but beneath the tactus things often get fuzzy.

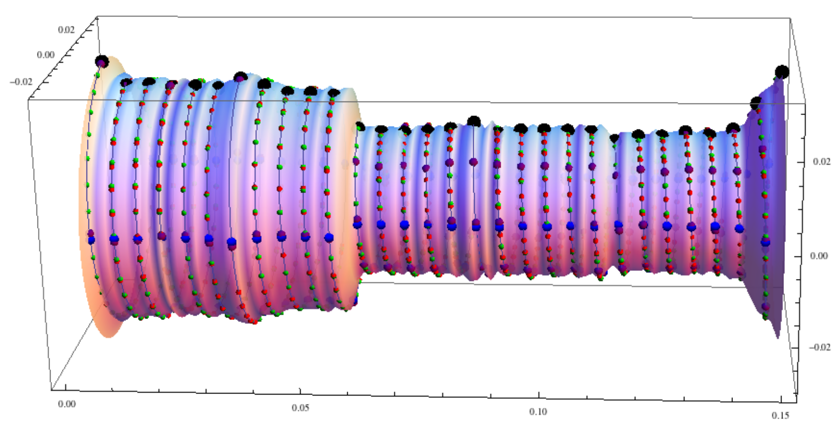
OPTIONS FOR THE LONGITUDINAL COMPONENT

Absolute track timing (or a scale thereof) is likely the most intuitive option for the z axis, although other options are possible. In the examples above, and in Example 12 below, z represents track timing scaled to 0.25. Alternatively, we can use the timing of one instrument as the cycle frame (c) while spacing those of another instrument equidistantly along in the z axis. In Example 13 the cycle frame (c) is based upon the *calung* while the gong is placed equidistantly along the z axis. This approach may best be used to describe musical experiences in which the regularity of the cycle is felt to overwhelm the flux of the material within it and may align with some "ethno-theoretical" descriptions of musical experience. Such an approach also facilitates the practical comparison of onsets between cycles by allowing us to either view or print each cycle as an equidistant "slice." When cycles occur in tight temporal succession the spheres representing previous and subsequent cycles may "bleed in" to the cycle with which we are concerned, obscuring our view.

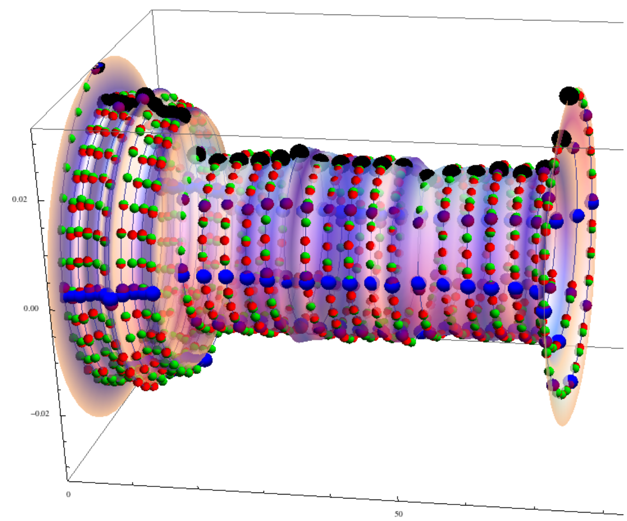
Example 12. *Z* axis as track timing, scaled to 0.25*; calung* radius, $k = 0.25$, $s = 3$;
Bapang Selisir, Pinda village



Example 13. *Z* axis as equidistant gong cycles, no scaling*; calung* radius, $k = 0.25$, $s = 3$;
Bapang Selisir, Pinda village



Example 14. *Z* axis in fluid dynamic model, no scaling; *calung* radius, $k = 0.25$, $s = 3$; *Bapang Selisir,* Pinda village

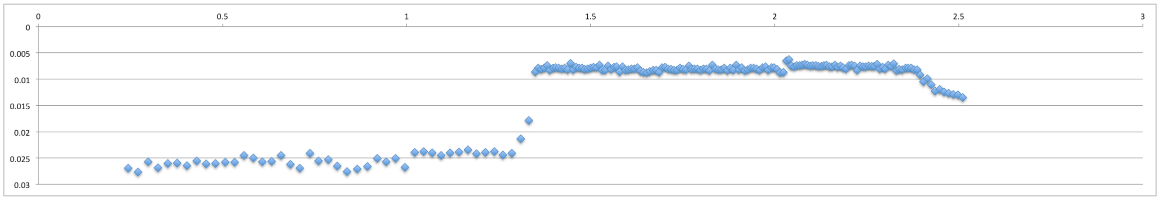


A more complex, and admittedly exotic, approach would be to space the secondary instrument "fluidly" along the *z* axis, placing its onsets further apart as they accelerate, as though forced through a narrower channel. Example 14 illustrates this fluid dynamic approach in which the cycle frame (*c*) is again based on the *calung* and the gong serves as the secondary instrument. This approach may best be used to describe musical experiences in which a sense of cycle as a volume overrides the sense of linear or clock time.

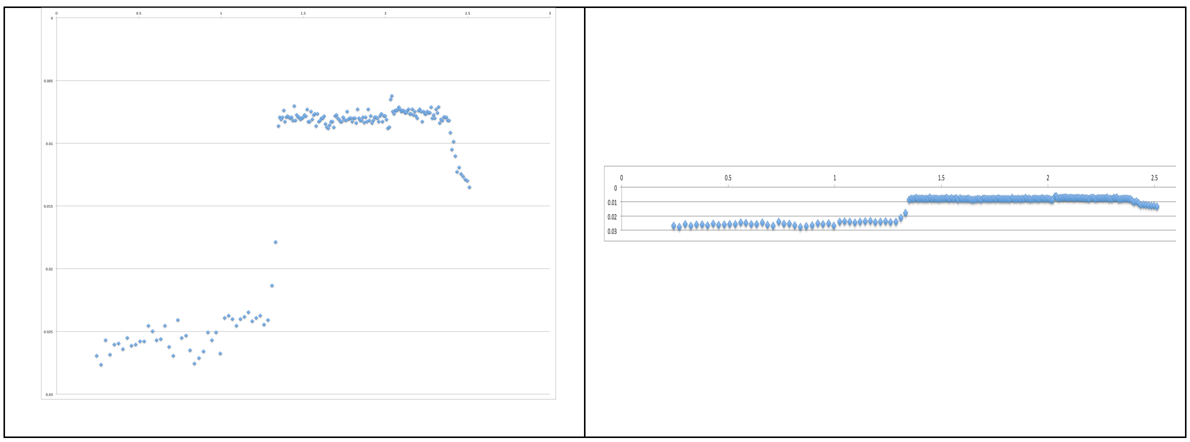
A NOTE ON SCALING AND PERCEPTION

Consider again the 2D representation of the *kajar* onsets for the Pinda performance of *Bapang Selisir,* repeated below in Example 15, and compare it to the two representations in Example 16. The representation on the left side of Example 16 appears to depict a comparatively more dramatic performance, with the erratic tempo in the first half and the sharp increase in the second half, that on the right a more monotone,

Example 15. Two-dimensional representation of *kajar* onsets; *Bapang Selisir,* Pinda village



Example 16. Scaling effects

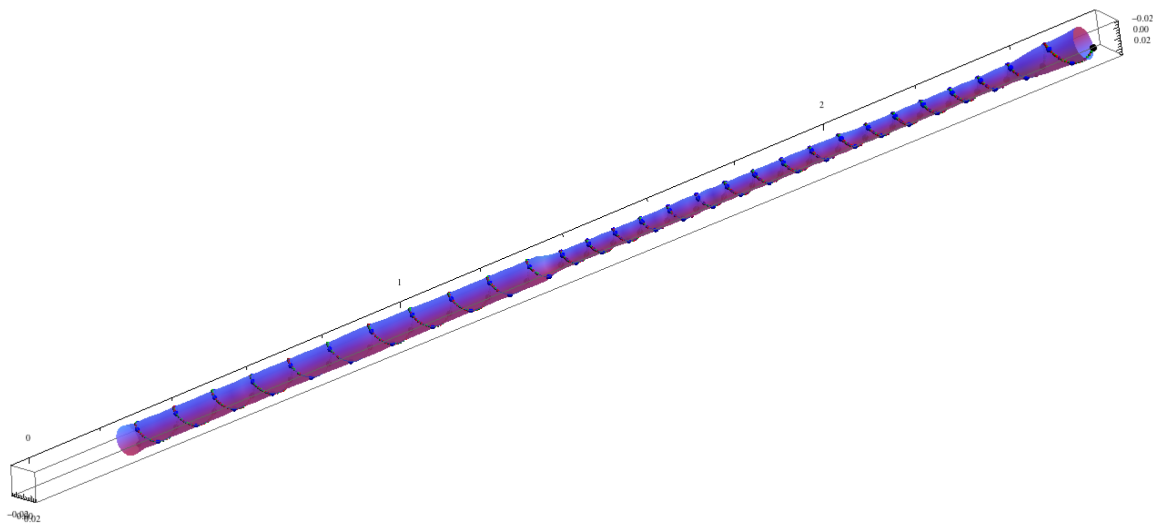


metronomic performance. Of course, Examples 15 and 16 represent the very same data. Why do we format and scale our visualizations the way we do?[6] If the data are all the same and the scaling transformations clearly stated and uniformly applied, why do we prefer one representation (often very strongly) over another? Why does Example 15 "feel right" to me in a way that neither graph in Example 16 does? Why do I find the un-scaled representation of Pinda's performance of *Bapang Selisir,* as pictured in Example 17, awkward to manipulate and difficult to read?

Representations of data are routed through perception, just as the data itself is. Unless we are trying purposefully to mislead, we represent data in ways that resonate with

[6] Tufte (2001) presents the authoritative analysis of this question.

Example 17. *Bapang Selisir,* Pinda village (no scaling)



the particular scales and magnitudes of human sensual and cognitive systems. The representation in Example 15 *looks like my experience of playing the music feels.* The tempo in the first section feels fluid to me, not rigidly metronomic as suggested by the graph on the right of Example 16. But neither does it feel erratic, as suggested by the graph on the left side. The vertical scaling of Example 15 aligns more closely with the vertical difference needed to account for meaningful temporal flux with respect to human time perception. Vertical expansion, as seen on the right of Example 16, raises the "temporal noise" of variations below the level of human perception to become apparently meaningful distinctions. The graph on the left of Example 16 might better represent a mouse's experience of the performance, that on the right an elephant's. Which representation we choose has as much to do with the technological conventions and limitations of our discipline as it does our physical experience of sound. A 2D vertical representation of onset timing aligns with particular cross-domain experiences of music;

we may feel ourselves "falling" in vertical space, towards the gong, as if descending a long wave. We may alternately experience music through a sense of revolution and volume, in which case a helical representation is more appropriate. In this way visualizing musical data is similar to ethnography; there are no absolute, neutral or omniscient perspectives.

REPRESENTING PHRASE AND FORM

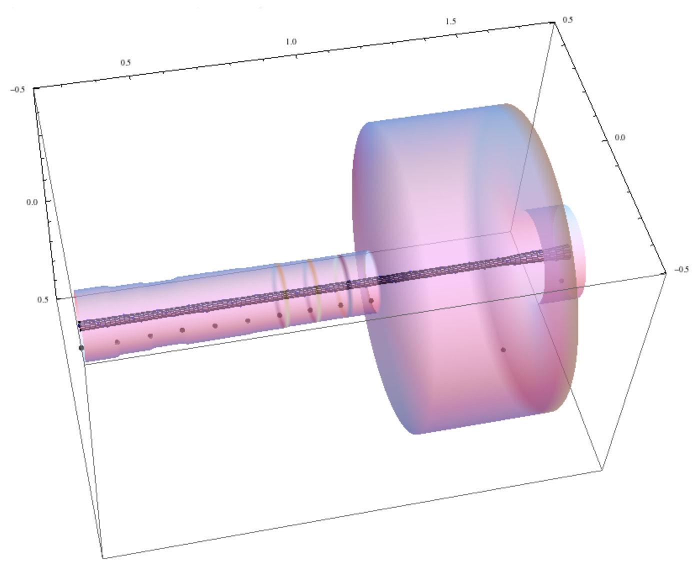
Form as a Virtual Space/Volume

We may experience formal divisions within music as virtual spaces with particular volumes.[7] When describing gamelan music this presents an obvious analog to the instruments themselves in which the volume of an instrument often correlates to the volume of time it outlines within the musical structure. The large gong holds (and moves) more air than the *kempur*, and it more than the *kajar. Jegogan* resonators activate more air than the *penyacah, calung,* or *gangsa.*

In the Pengosekan (Cudamani) performance of *Bapang Selisir,* the musicians included an extended 32-pulse melodic interlude (*ngelik*) toward the end of the performance. The interlude is widely known. In some villages the gong player will simply continue the 8-pulse cycle throughout the longer melody; in others and in Pengosekan, the gong player sits out during this section, functionally shifting from an 8-pulse cycle, once to a 32-pulse cycle, and back again to 8. This type of structure can be represented, as in Example 18, by nesting helices inside each other, one for each periodic layer/instrument.

[7] For a critique of the idea of music's spatiality, see Carpenter (1967).

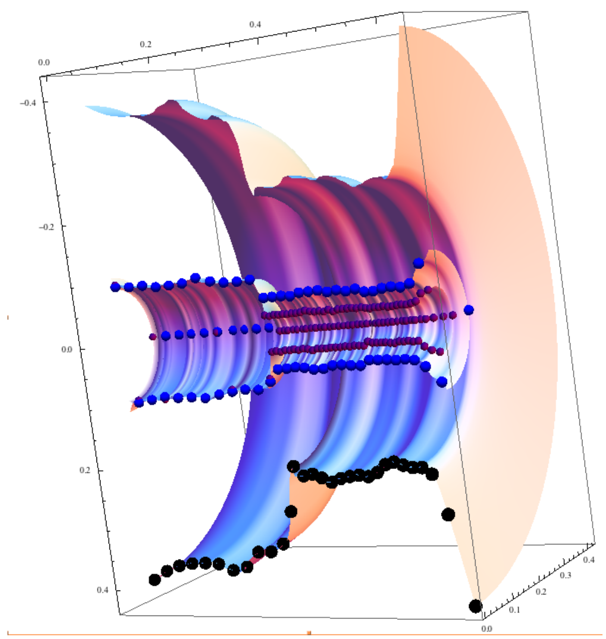
Example 18. Form as nested shells: *kajar* helix within *gong* helix, picturing extended 32-pulse interlude at the end; $k = 0.25$, $s = 3$; *Bapang Selisir,* Pengosekan village (Cudamani)



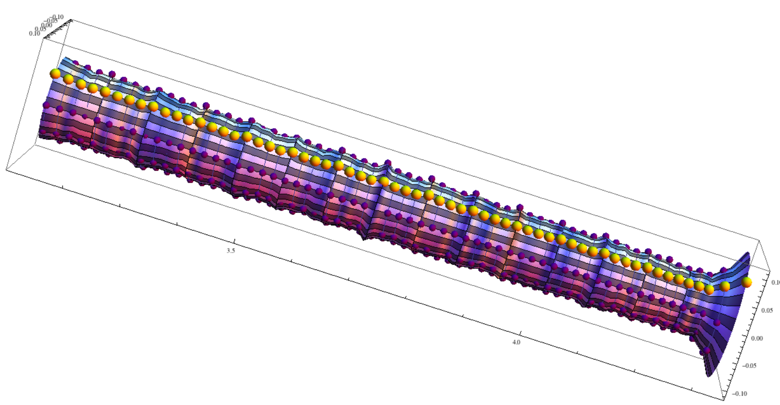
Drawing surfaces upon these helices produces the image of nesting Matryoshka dolls of musical time.

This technique also allows us to visualize shifting relative densities between periodic layers. In Pinda's performance the *kajar* musician shifts in density from four onsets per cycle in the slower first half, to eight onsets per cycle in the double time. This is illustrated in Example 19, where half of each shell has been sheared away. The representation of form as a volume—a musical space we feel we are "in"—may be most salient for the musicians and for listeners who have previously experienced the music. Only those listeners already know if we are entering a big or small space. In other contexts it may make more sense to represent form as a differently colored shell, or extended melody in its second half that is four gong cycles long. While no special instrument is reserved to audibly announce the ending (and beginning) of the overall melodic form, this moment is dramatically indicated by temporal fluctuations, primarily a swelling decrescendo toward the final gong statement. This is illustrated in Example 20 by an alternating grid pattern.

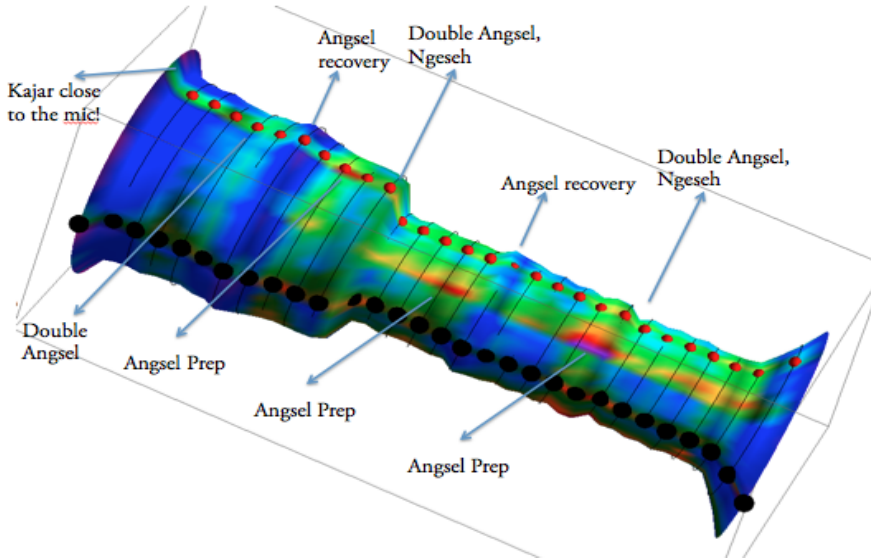
Example 19. Density shifts as nested shells: *kajar* (purple) within *calung* (blue) within *gong* (black) helix; $k = 0.25$, $s = 3$; *Bapang Selisir,* Pengosekan village (Cudamani)



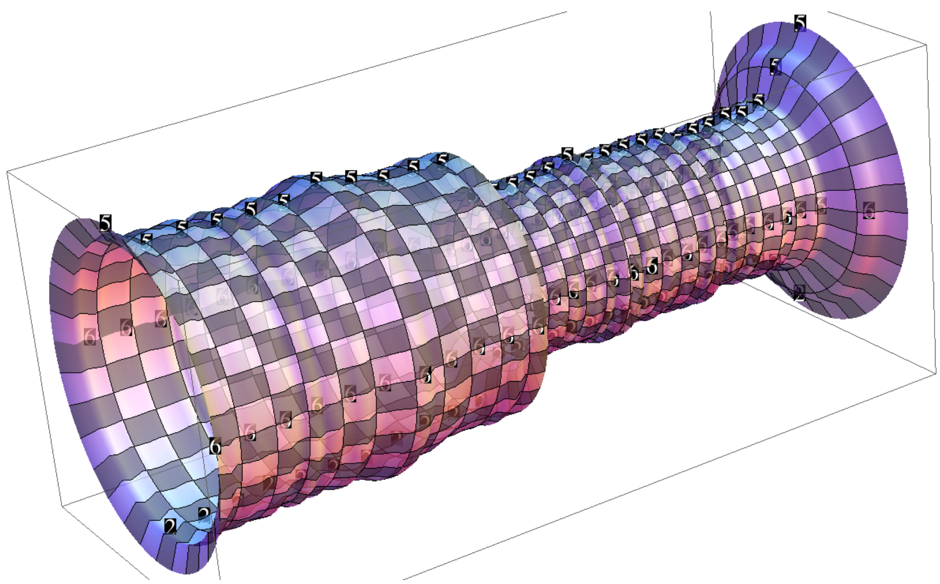
Example 20. Representing multi-*gongan* formal structure with alternating surface grid: tactus (purple), gong (yellow), tactus radius, $k = 0.25$, $s = 3$; *Tabuh Gari,* second half; STSI Bali, commercial recording



Example 21. Surface as amplitude; blue = quieter, red = louder; showing *calung* (red) and gong (black) onsets; *calung* radius, $k = 0.25$, $s = 3$; *Bapang Selisir,* Pinda village



Example 22. Including pitch information; *calung* onset pitches; *calung* radius, $k = 0.25$, $s = 3$; *Bapang Selisir,* Pinda village



FURTHER VISUALIZATION OPTIONS

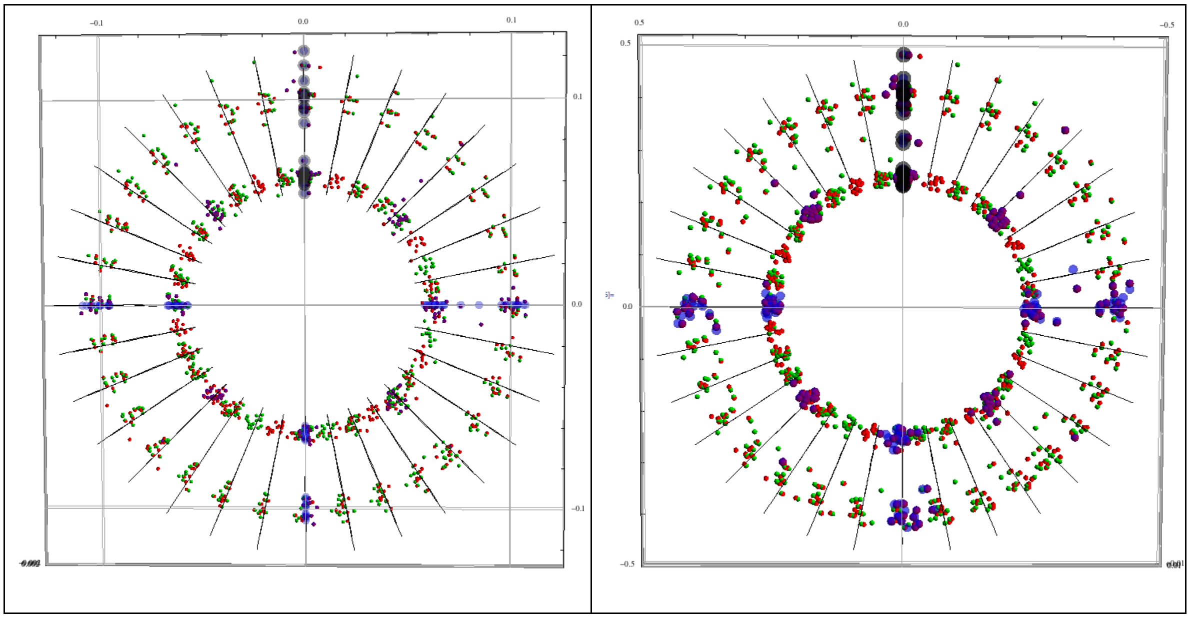
Although the code developed for the present project is primarily concerned with the analysis and visualization of timing information, it is also possible to include dynamic and pitch data. In Example 21 the amplitude levels of the room recording of Pinda's performance is wrapped along the surface to visualize the interaction of timing, gestural, and dynamic aspects. Discrete pitch information can be drawn on onsets as numbers, as illustrated in Example 22. This works well with the numerical cipher notation systems favored for gamelan and many Southeast Asian traditions. Alternately such indications can refer to Western scale degrees.

MICROTIMING

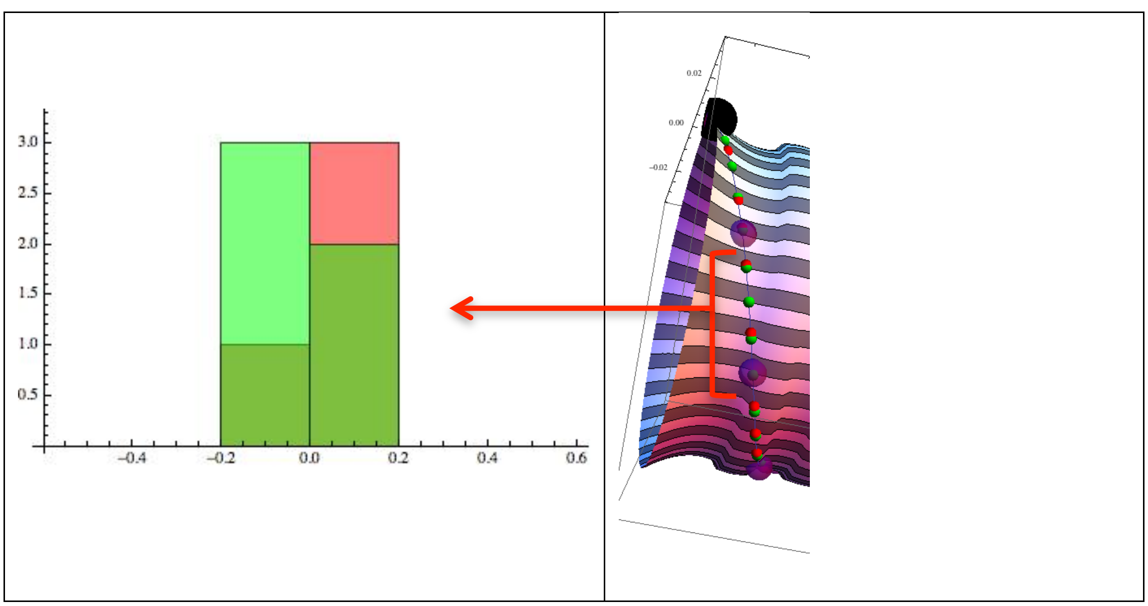
When the grid is included, looking down the length of the helix functions as a cumulative histogram of microtiming.[8] In Example 23 we can clearly see the offset between the rational subdivisions implied by the radial instrument and the actual placement of the onsets of other instruments. The left-hand image illustrates the *calung* perspective helix, the right-hand image the gong perspective helix, displaying more offset. We can compare the structure of microtiming between gestures within or across performances by developing a simple histogram in which onsets are placed in a negative bin if they occur 50% or less after the closest equidistant subdivision (suggested by the fastest periodic layer, i.e., the *kajar* at the quarter-note level) and thus dragging;[9] and placing onsets in a positive bin if they occur 50% or more before the closest subdivision,

[8] This approach is similar to that demonstrated in Benadon (2007). However, Benadon's subdivisions vary radially to demonstrate onset relationship to multiple grids, and onsets are drawn counterclockwise. By contrast, the subdivisions above are drawn angularly and clockwise with respect to a single grid.
[9] Onsets beyond 50% would be considered rushing towards the subsequent subdivision.

Example 23. Looking down the length of the helix to visualize microtiming; *calung* radius (left), gong radius (right), $k = 0.25$, $s = 3$; *Bapang Selisir,* Pinda village



Example 24. Microtiming histogram of *polos* (red) and *sangsih* (green) onsets with respect to *kajar* (purple); beats 2 to 3, gong 1; no scaling; *kajar* radius; *Bapang Selisir,* Perean village



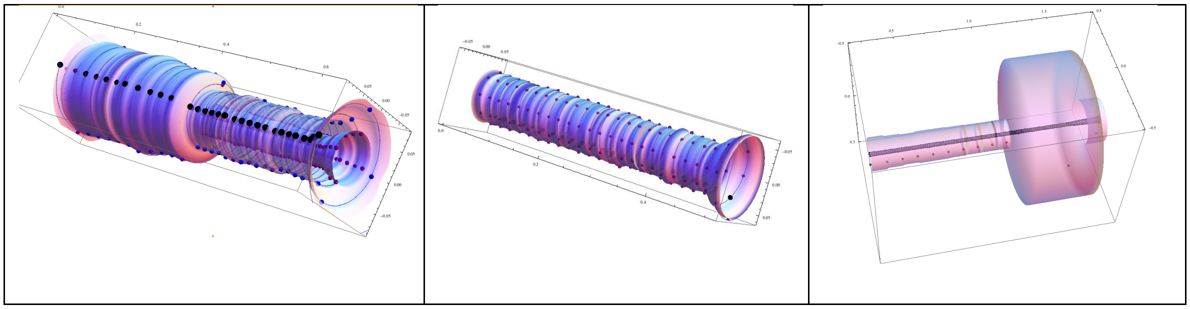
and thus rushing.[10] This is visualized in Example 24 by considering just the *gangsa* onsets between second and third *kajar* onsets in the first gong of the performance of *Bapang Selisir* by the Perean ensemble. In both the histogram and the helix, *polos* are represented by red, *sangsih* by green. The Y axis represents the number of onsets in that bin. Within the time frame specified we have four *polos* onsets, five *sangsih* onsets. Here we see that three sangsih onsets were rushing while three were late, and that three polos onsets were rushing while one was late. In the histogram darker hues indicate bins occupied by both *sangsih* and *polos* onsets. The grid divides the pulse into thirty-second rather than sixteenth notes to illustrate the relationship between *gangsa* onsets and their placement in the histogram.

COMPARING THE STRUCTURE OF MUSICAL TIME: PINDA, PEREAN, AND PENGOSEKAN

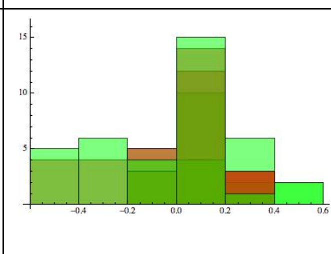
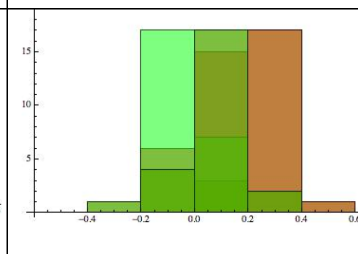
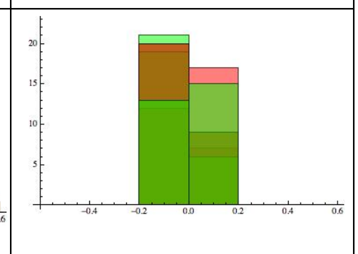
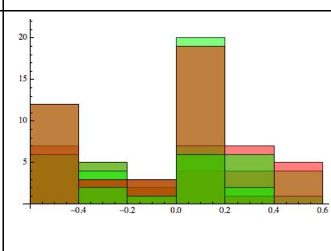
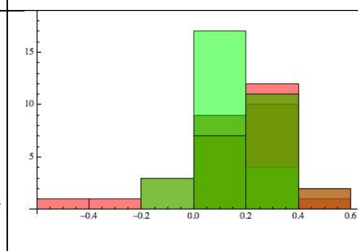
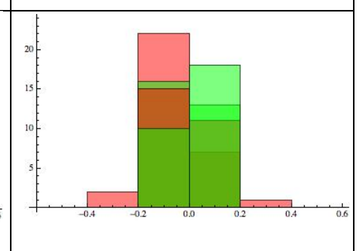
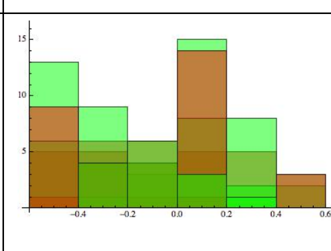
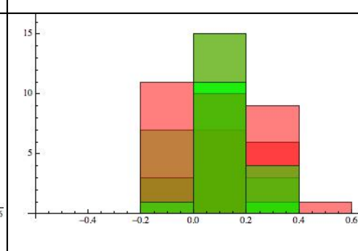
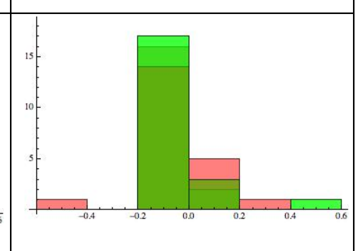
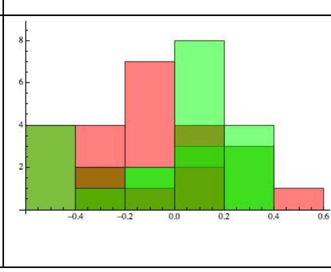
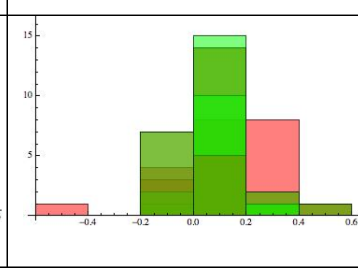
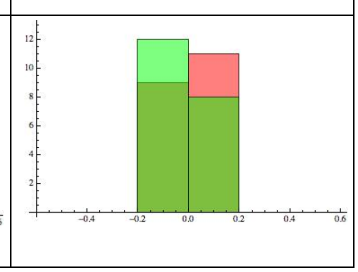
We now have the tools to broadly compare the structure of musical time (both piece-length and microtiming) in performances of *Bapang Selisir* by the ensembles from Pinda, Perean, and Pengosekan.[11] Example 25 compares the helical shapes produced by the three performances, each temporally complex, nuanced, and distinctive. The histograms presented in Example 26 compare the microtiming of *polos* and *sangsih* onsets (relative to *kajar* onsets) by structural gesture. Each histogram is based on three sample gestures from each performance. An animation of the histogram for each gong cycle within a performance demonstrates the ways in which microtiming contributes to the structure of gestures throughout a piece. Click here for an animation for the histograms of the Pinda

[10] Onsets prior to 50% would be considered dragging after the previous subdivision.
[11] Click here for links to room recordings of each performance.

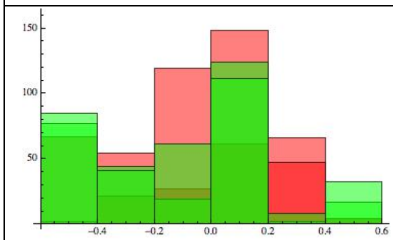
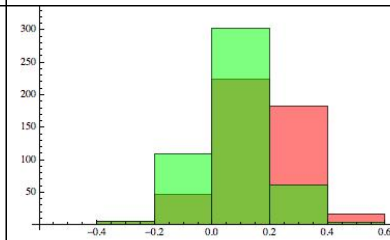
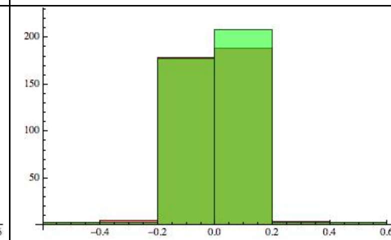
Example 25. Piece-length structuring of time; *Bapang Selisir* as performed by ensembles from Pinda (left), Perean (middle) and Pengosekan (right); $k = 0.25$, $s = 3$



Example 26. Comparing microtiming of *polos* and *sangsih* onsets by gesture; *Bapang Selisir,* Pinda, Perean, Pengosekan

Gesture Type	Pinda	Perean	Pengosekan (Cudamani)
Stable			
Angsel Prep			
Angsel			
Angsel Recovery			

Example 27. Comparing microtiming of *polos* and *sangsih* onsets, full performances
Bapang Selisir, Pinda, Perean, Pengosekan

Pinda	Perean	Pengosekan
		

performance. Example 27 summarizes the microtiming data by displaying onsets for entire performances.

Caveat Lector

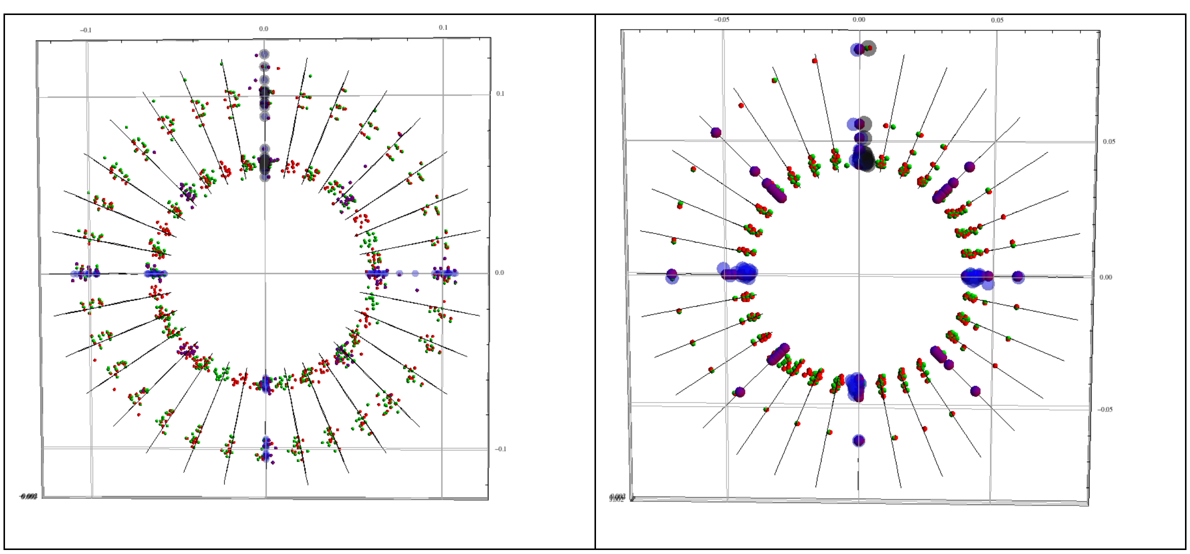
To borrow a phrase from the physicist Wolfgang Pauli, it would be "not even wrong" to equate the machine measurement and representation of onsets performed by a limited number of musicians in single performances to the subjective evaluation of style based upon the embodied experience of countless "onsets" and thousands of performances over a lifetime. The analyses above describe only single performances within a very fluid and open performance culture; they do not capture the full range of expression possible by any ensemble; different day, different performance. To do full justice to the musicians and the village styles they represent, we would need a larger dataset as well as combined histograms and helical shapes from many performances. Only then could we begin to empirically characterize the experiences and evaluations of informed listeners.

MUSICAL TIME AS AN ICON OF COMMUNITY

The representations above might be best described as *visual heuristics* that can help us *approach* an understanding of the implicit nature of temporality as an element of style and communal identity, elements which are often obscured or ignored by the technology previously employed to analyze Balinese music. However, the visualizations above suggest that the Pengosekan (Cudamani) players play with much more compactness (closer together) than those of either Perean or Pinda performers. This would seem ironic considering that these are the young musicians who evinced a nostalgia for the bygone communitas that they themselves identified in the styles of Pinda and Perean (as well as Gladag and Jagaraga). Consider a cumulative view of the Cudamani musicians' onsets, comparing them to those of Pinda, as illustrated in Example 28.

Although the sample sizes are different—the longer Pinda performance includes 1,360 onsets to Cudamani's 971—the trend is clear. The Cudamani musicians are more

Example 28. Cumulative view of microtiming. Pinda (left), Pengosekan-Cudamani (right); *kajar* radius; *Bapang Selisir*



"squarely on the grid," so to speak, while Pinda's musicians approach more closely Benadon's concept of the "gridless beat." The self-described moderns adhere more closely to what we would call the "rational" equidistant subdivisions of the pulse as suggested by standard Western staff notation. To what extent are musicians' characterizations of the iconic ensembles of *le temps perdu* real, remembered, or nostalgic? The median age of the Pinda, Perean, Gladag and Jagaraga ensembles was younger when they became famous across the island decades ago. Were we able to travel back in time to conduct these recordings, would their performances adhere more closely to Cudamani's? Do the representations above reflect arthritic wrists as much as temporal structuring as an icon of local musical identity?

This could only partly be true as each ensemble has fostered a regular influx of young musicians who, according to the older masters, similarly embodied the distinctive village style. Are the more subtle cultural changes alluded to by some musicians illustrated within these graphs? Is the increasing experience of homogenous and striated space and time—metronomic Western style popular music, clocktime over ceremonial time, the grids of the city street and Mercator map, the rational slices of equal-tempered tuning (ascendant in Bali), and so on—exerting a homological manifestation in the subtle structuring of musical time in contemporary interpretations of traditional Balinese gamelan music? Is this phenomenon homogenizing local musical cultures globally? If this is the case, then music's association with place necessarily fades with the spread of "universal" and rational modes of ordering elements such as pitch, time, etc. As many philosophers have argued, the rise of the homogenous, rational slices of the grid facilitates surveillance, ordering, monetization and control.[12]

Benadon (2009, 158) summarizes Sakai's studies on the cognitive effects of the musical grid:

> Katasuyuki Sakai and his colleagues used brain imaging to point to two distinctive neural representations of rhythm: those that employ simple integer ratios and those that do not (Sakai, et al. 1999). Complex ratios elicited neural activation in areas involved in higher order cognitive processes, such as attention and working memory. This was not the case with simple ratios. Since these do not require explicit representation of individual time intervals, they lend themselves to hierarchical organization, thus placing less of a tax on computational resources.

Simple integer rhythmic ratios align to the grid and apparently allow our cognitive systems to go into a kind of autopilot. They implicate a homogenized internal experience of music as well as a homogenization of space by eroding the complex structures of musical timing that have long been embodied by local groove. Although the Cudamani ensemble is renowned and respected island-wide as one of Bali's leading ensembles, an elder musician once described the ensemble to me as "too good, too clean." Ironically, in some contexts the management of a *diversity* of microtiming intervals may produce a sense of togetherness more so than a unified adherence to a grid. Like many jazz ensembles, Pinda's fame might be partially traced to their forging a collective artistic identity through the virtuosic use of apparently irrational microtiming intervals.

THREE ADDITIONAL EXAMPLES

Lost and Lookin'

Some commercial recordings are sufficiently sparse to allow for onset detection through a combination of spectral and beat detection plugins in programs such as Sonic

[12] See Janier (2013) and Abrams (1996) for commentary on these topics.

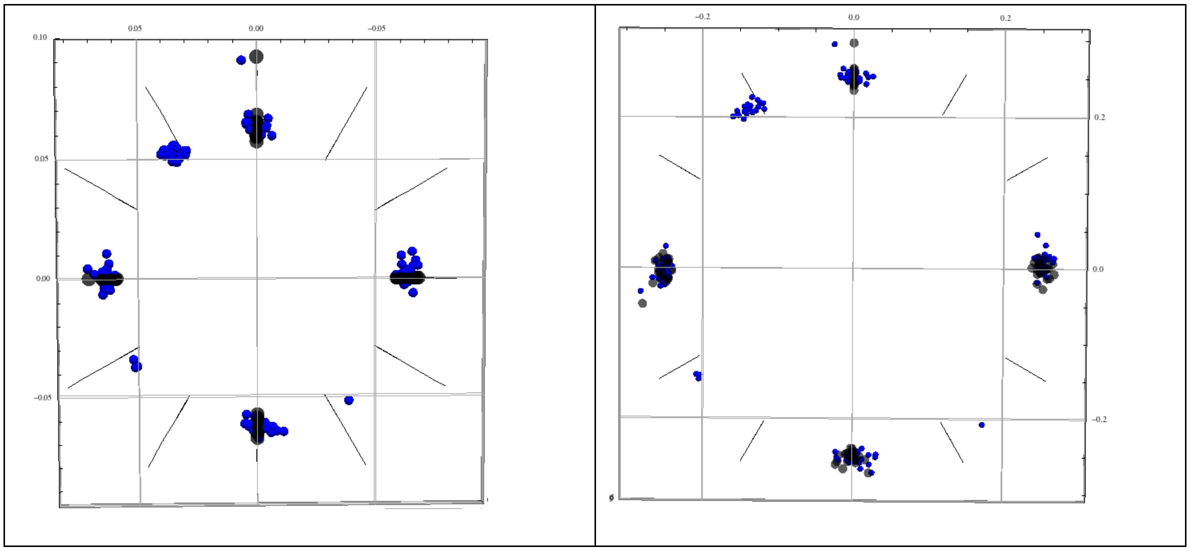
Visualizer. Sam Cooke's *Lost and Lookin'* (1963)[13] includes only double bass, ride cymbal, and Cooke's voice. The simple groove, with the bassist laying down a solid quarter note and the drummer playing a minimalist "swing" shuffle on the ride cymbal, is made profound by the complex, expressive interaction of onsets, visualized cumulatively in Example 29. Twelve grid subdivisions display the equidistant position of quarter-note triplets, mapping a deadpan swing. Ride-cymbal onsets anticipating the pulse (landing overwhelmingly before beat 1) deviate significantly from the square placement suggested by the equidistant grid, aligning closer to the quintuplet interpretation of swing groove at slow tempos as identified by many jazz scholars (cf. Benadon 2009). We can consider the rich "participatory discrepancies" (Keil 1995) between the bass and drums from the perspective of the quarter-note (bass) cyclical perspective (left-hand graph) or the fluid placement of the quarter note within the 4/4 meter by assigning the first bass note as the metrical (n = 1) perspective. Viewing the helical structure in Example 30 reveals that even the ostensibly metronomic grooves of Western popular music can, when performed by musicians interacting in live recording contexts, flow with expressive timing. Here Cooke's alternate verse in the second half is indicated by a differently hued surface.

Gavotte en Rondeaux

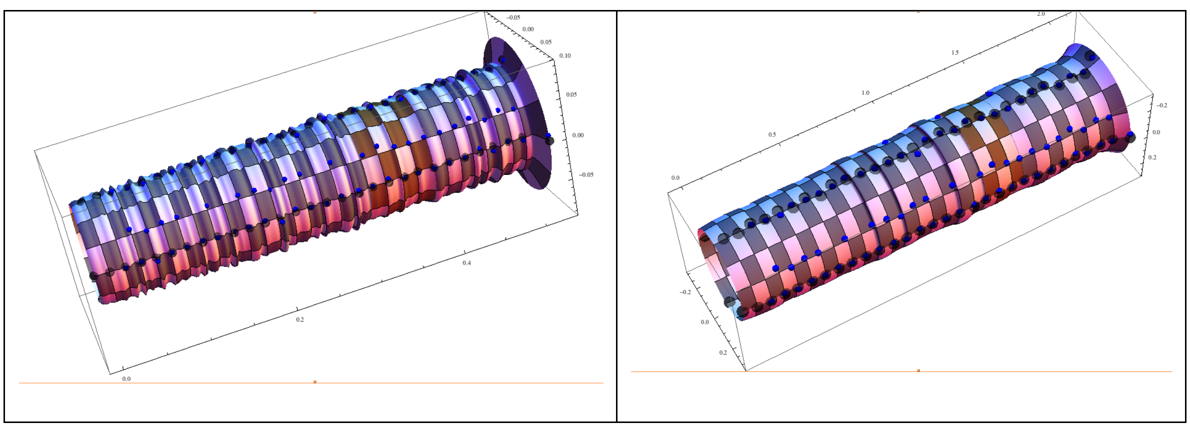
From Bach's Partita No. 3 in E major for solo violin, the gavotte in rondo as performed by Arthur Grumiaux[14] ripples with expressive timing wholly absent from the original score and which would be forbiddingly difficult to capture within a standard

[13] Released in the US on the album *Night Beat,* RCA 2709. Clifford Hills (bass), Edward Hall (drums).
[14] Philips 464 673-2, recorded between 1960-1. Onsets were determined using the Aubio onset detector in Sonic Visualizer.

Example 29. Cumulative view of bass (black) and ride cymbal (blue) onsets; bass radius (left), metrical radius (right); Sam Cooke, *Lost and Lookin'* (1963)

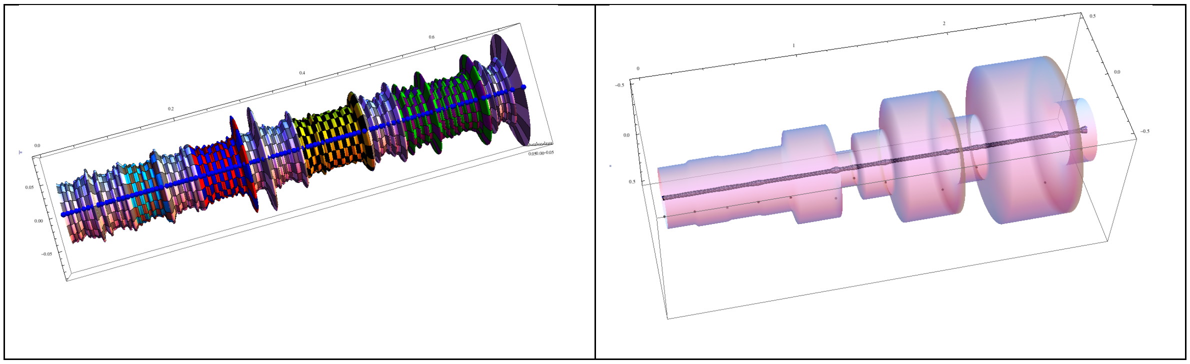


Example 30. Helical view of bass (black) and ride cymbal (blue) onsets; bass radius (left), metrical radius (right); Sam Cooke, *Lost and Lookin'* (1963); $k = 0.25$, $s = 3$



Western transcription. The rondo form is indicated in the left hand graph of Example 31 by alternating grid coloring, illustrating the A,A,B,A,C,A,A,D,A,E,A structure. Temporal swells coincide with most structural endings, with interesting exceptions. The right-hand graph renders the same form as volume.

Example 31. Tactus radius; colored grid illustrating form (left), k = 0.25, s = 3;
form represented as volume (right), no scaling.
Bach, *Gavotte en Rondeaux* from Partita in E Major, BWV 1006, performed by Arthur Grumiaux

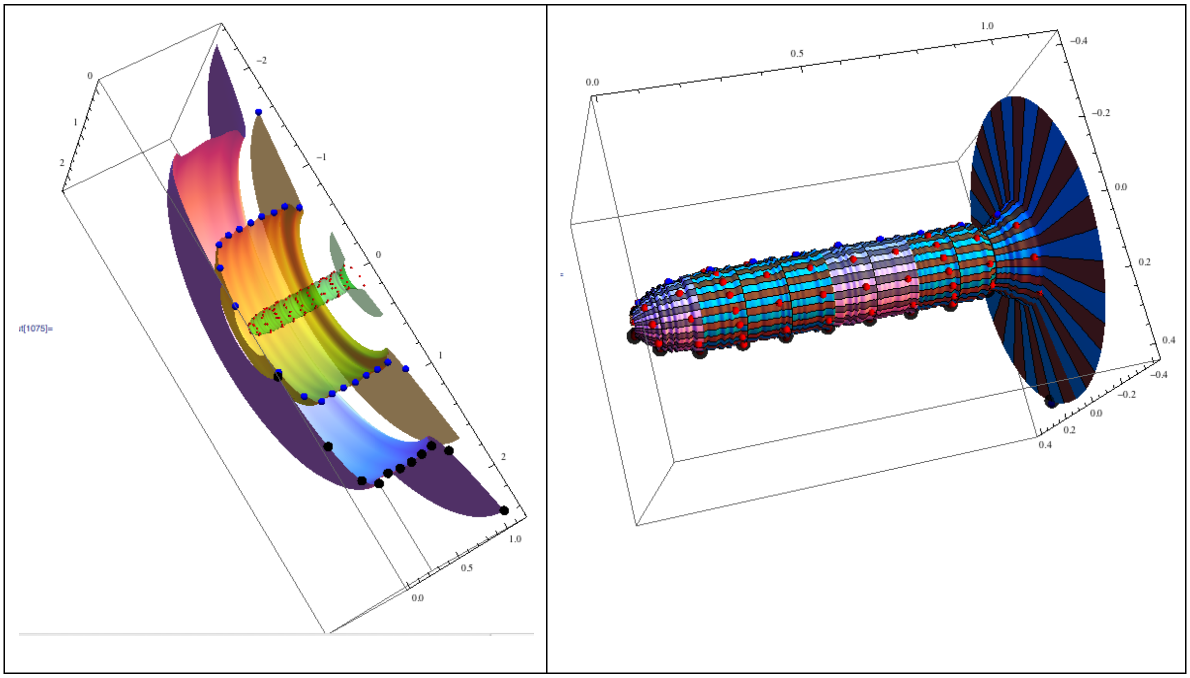


Puspawarna

The final example illustrates the temporal structuring of a performance of the classical central Javanese gamelan composition *Puspawarna,* performed by the musicians of the Paku Alaman court in Yogyakarta in 1971.[15] Example 32 illustrates the 16-pulse *ketawang* form outlined by a single gong stroke (black) and bisected by two *kenong* chime strokes (blue), with one coinciding with the large gong. The *balungan,* or abstracted melodic frame, occurs at an irregular rate at the level of the tactus (red). The left-hand graph illustrates the nesting surfaces of the gong (pink surface), *kenong* (yellow surface) and *balungan* (green surface). The right-hand graph draws all onsets on a helix based upon the tactus frame. This helical form is common to many Central Javanese gamelan compositions in which the form opens into a fast tempo, quickly slowing down and shifting density (the *irama* change) into a more stable tempo typically divided into four subdivisions (*irama dadi*) before either slowing to the final gong (preceded by an increase

[15] Directed by K.R.T. Wasitodiningrat. From the album *Javanese Court Gamelan*, Nonesuch Explorer Series (79719). Onsets were produced using the tempo tap feature in Sonic Visualizer.

Example 32. Helical view of *Puspawarna*; nested shells illustrating gong (red), *kenong* (yellow) and *balungan* (green) helices (left), no scaling; tactus radius indicating formal divisions as a colored grid (right), k =0.25, s = 3



in tempo in the penultimate *gongan*), or downshifting through subsequent *irama* levels. The alternating grid colors in the graph indicate the *ompak* and *ngelik* formal divisions, pink/grey and cyan/brown respectively. I will refrain from speculating on the potential iconicity of the helix pictured on the right-hand side. However, considering the venerable tradition of double entendre in Javanese music, I highly doubt any of the musicians would be offended.

3D PRINTING

Each of the 3D models discussed in the present paper can be easily exported in a variety of formats (.stl, .wrl, .3ds, .obj) for physical printing, either as solid objects or shells (see Photo 2). Interacting with the material representation of music leverages human

Photo 2. 3D printed shapes based upon examples described above

synesthetic audition. While a 2D representation of onsets is preferred when it is desirable to visually perceive an entire performance simultaneously, following the onsets around a helix on a physical shape in one's hand is more reminiscent of the temporal unfolding of our experience of music itself. Physical printing takes advantage of the human ability to rapidly make sense of visual and tactile information and allows for the quick comparison of similar phenomena. Comparing the piece-length temporality and microtiming between different performances of the same work is more practical through "playing" with objects in one's hand than in attempting to zoom and manipulate multiple 3D representations on a single screen. My own anecdotal experience suggests that the physical manifestations (more than on-screen representations) of theoretical models encourage interactive self-learning amongst students.

LIMITS OF THE MODEL

The model proposed in this paper is limited to periodic musics incorporating consistent metrical structures. Musics incorporating frequent rhythmic irregularities cannot be represented (at the metrical frame). Unlike 2D representations (such as Example 4), all data points cannot be viewed simultaneously in the helical model. The model is not practical for extensive pitch-based analysis and cannot accommodate continuous pitch information. Note length is not accounted for in this model, which was designed primarily to accommodate gamelan musics in which note onset takes analytical priority because of the steep dynamic envelopes of percussive instruments.

DOCUMENTATION

Click here for access to free downloads of all code, source data, and (non-commercial) recordings referenced in this document.

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